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EFFECT OF VIBRATION ON A NON-HOMOGENEOUS ORTHOTROPIC RECTANGULAR PLATE WITH LINEARLY VARYING TEMPERATURE AND CIRCULARLY VARYING THICKNESS

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ABSTRACT

The vibration properties of a non-homogeneous orthotropic rectangular plate with circularly variable thickness and temperature are investigated in this work. To accommodate for non-homogeneity, the plate’s material density is considered to fluctuate linearly in the x-direction. The general differential equation governing the orthotropic rectangular plate's behavior is solved using the Rayleigh-Ritz Method. The resultant equations provide the frequencies for the first two vibration modes. Under clamped boundary conditions, a number of structural characteristics are examined, such as aspect ratio, non-homogeneity, thermal gradient and taper constant. All calculations are performed using MATLAB, and the results are presented through graphs and tables.

KEY WORDS: Orthotropic rectangular plate, taper parameter, circular thickness variation, clamped boundary condition and linear density.

1. INTRODUCTION

Vibrations in machinery play a crucial role in modern technology. It is associated with daily aspects of life which include our working and living life style. This vibration is used in airway, roadway, marinas and other source of technical fields. The effect on vibration plays an important role weather it is a large or small. Due to the effect on vibration, temperature and density of a machine vary with deflection in the wings of helicopter and strain the structure of the transport vehicles. The effect of temperature and density in vibration with high level thickness material can be used in various field engineering, marines, power plants, aeronautical, chemical such as nuclear. Non homogeneity in orthotropic rectangular plate in visco elastic material plays an important role in gas turbines, space craft and engineering field. In orthotropic material a lot of work is done by using the different variation in plates. But a less work has done in orthotropic plate with effect of circular variation which includes the linear density. In the present day all the machine variation have the common design. We need such type of structure which is better in designing and also easy to analyzing. To get the better strength and durability, we need to develop the design with low expense and low in weight. In the field of space and technology large machine are design in such a way that it can avoid the harmful vibration which saves us from earthquake, electronic components and smokestacks. In orthotropic rectangular plate needs a controlling vibration effect which is due to the effect of temperature and density helps in building construction, paper industry and space shuttles. Thus we have to maintain the vibration of the plate and it should be free from unwanted and harmful elements which cause the fatigues.

Many researchers studied different type of plates such as elliptical, rectangular and circular with different thickness and temperature. Plates with different type of thickness variation and boundary condition are analyzed [1]. Reference is made to a viscoelastic rectangular plate with thickness that follows a parabolic distribution. While significant research exists on the elastic and inelastic behavior of materials, limited studies have focused on viscoelastic bodies with varying bearing thickness. This study examines how the taper constant influences the free vibration of a clamped viscoelastic rectangular plate featuring featuring a parabolic variation in thickness [2]. The effect of thermal gradient on vibration of clamped boundary condition with thickness variation varies linear and parabolic analyzed [3]. This research was conducted to minimize the natural frequencies of the plate and examine how changes in plate parameters impact these values. To support the study’s objective, numerical examples are provided along with a comparative evaluation against existing results. Two dimensional circular thicknesses are obtained by using Ritz method in a rectangular plate [4]. An analysis of the forced vibration of a non-homogeneous rectangular plate with thickness that changes linearly is carried out using classical plate theory. The material is regarded

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as non-homogeneous due to a linear change in its density. To evaluate the structural response, approximate deflection of the plate under a uniformly distributed harmonic lateral load [5]. This study analyzed the non homogeneity of visco elastic material varying thickness with thermal effect [6]. The transverse vibration behavior of a rectangular plate with thickness variation is investigated under multiple boundary condition scenarios at its edges. The model captures thickness changes in two directions as the Cartesian product of linear variations along two adjacent sides, employing the Rayleigh-Ritz method with suitable trail functions and iterative successive approximations.. A comprehensive numerical analysis is conducted to calculate the first three natural frequencies [7]. The vibration of exponential variation in non homogeneity discussed with a rectangular plate [8]. Behavior of the vibration of clamped rectangular plate with linear thickness in both directions has been studied [9]. The study calculates the vibration properties of an orthotropic rectangular plate featuring thickness variation and subjected to temperature changes. The influence of thermal stresses and material anisotropy on the natural frequencies and mode shapes has been analyzed [10]. Using thin plate theory and two-dimensional viscoelastic constitutive equations, the governing differential equation is formulated for a viscoelastic plate with linear thickness variation and multiple through-cracks. Expression for the additional rotational angles caused by the cracks is also derived. The material is assumed to exhibit elastic behavior under dilation, while its distortion follows the Kelvin-Voigt model [11]. A double trigonometric series is used to derive an analytical expression for the characteristic equation of a clamped orthotropic rectangular plate with thickness variation in a single direction parallel to its sides. Utilizing the fundamental natural frequency obtained from this equation, a numerical formula is proposed to estimate the fundamental frequency of the clamped orthotropic plate [12]. An analytical approach based on superposition yields an accurate solution for the free vibration of a fully free orthotropic rectangular plate. For a particular case representing reinforced and composite like materials, eigen values for 12 vibration modes are provided across a wide spectrum of plate aspect ratios. Although it is not possible to present results for all potential scenarios, the data includes a broad and representative selection of cases [13]. This study gave an analysis to the vibrational of a visco-elastic parallelogram plate featuring a parabolic thickness profile [14]. Vibration analysis of a non-uniform, orthotropic rectangular plate with variable thickness is performed using the Rayleigh-Ritz method within the framework of classical plate theory. The approach involves applying two-dimensional orthogonal polynomials for boundary characteristics, produced through the Gram-Schmidt technique. The plate undergoes uniformly distributed in plane loading on opposite edges, featuring a clamped boundary on one side and a simply supported boundary on the other. The material non-homogeneity is attributed to a linear variation in both the elastic properties and density within the plane coordinates [15]. An analysis is performed on the transverse vibrations of an orthotropic, non-homogeneous viscoelastic circular plate featuring a radially parabolic bead thickness distribution. The non-homogeneity is modeled by assuming a linear variation in material density with respect to the radius. Using two-term deflection function is formulated through the Rayleigh-Ritz method [16]. Non-homogeneity along two adjoining edges is considered in the study of free vibration behavior in a thin rectangular plate. Linear variation is applied to both the modulus of elasticity and the density of the plate. Finite difference method has been applied [17]. To analyze the natural vibration behavior of clamped orthotropic rectangular plates, the extended Kantorovich method is applied. The employed dual iterative procedure converges quickly to accurate results. In the absence of exact analytical solutions for such plates, the obtained results show strong agreement with available data. The derived closed form expressions for natural modes and their corresponding frequencies are well suited for engineering analysis and practical applications [18]. With deep analysis to the orthotropic rectangular plate with two dimensional temperature and thickness variation has been analyzed [19]. The exact solution for free vibration of thin orthotropic rectangular plates studied [20]. Effect of the non-homogeneous rectangular plate with parabolically thickness in both direction and exponentially temperature distribution has been analyzed [21]. Researchers previously mentioned analyzed the vibration of orthotropic rectangular plates considering specific values for thickness and temperature. However, some work is done with linear temperature in both x and y dimensions, respectively and circular thickness in x-direction. Our current work's primary goal is to investigate the linearly varying temperature influence on the x-direction vibrations of an orthotropic rectangular plate with circular thickness subjected to clamped boundary conditions along all edges are examined.

By leveraging a deflection function within the Rayleigh-Ritz framework; the motion equation is resolved, leading to the calculation of frequency parameters for the first two vibration modes. Density varies linear with circular thickness in one dimension and temperature in both directions. Thermal gradient is denoted by **(A)** and taper parameter with **(B)**. Also non

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homogeneity is denoted by $(\mathbf{m_1})$. Graphical and tabular data present the impact of non-homogeneity, aspect ratio, temperature gradient and the taper constant.

The differential equation of motion of visco-elastic orthotropic rectangular plate may be written as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 W}{\partial t^2}\,. \quad (1)$$

Here M_x, M_y and M_{xy} represent the bending moments and the twisting moment per unit length acting on the plate in the x-direction and y-direction, respectively. ρ be the material's mass density, h is the plate's thickness, and w indicates its displacement as a function of time t. The terms M_x, M_y and M_{xy} are given as:

$$M_x = -\check{D}\, D_1\left[\frac{\partial^2 W}{\partial x^2} + v\frac{\partial^2 W}{\partial y^2}\right]$$

$$M_y = -\check{D}\, D_1\left[\frac{\partial^2 W}{\partial y^2} + v\frac{\partial^2 W}{\partial x^2}\right]$$

$$M_{yx} = \check{D}\, D_1\,(1-v)\frac{\partial^2 W}{\partial y\,\partial x}\,. \quad (2)$$

where $\tilde{D}$ is the representation for visco-elastic operator.

$$D_x = \frac{E_x l^3}{12(1-v_x v_y)}\ ,\ D_y = \frac{E_y l^3}{12(1-v_x v_y)}\ ,$$

$$D_{xy} = \frac{G_{xy} l^3}{12(1-v_x v_y)}\ . \quad (3)$$

$D_1 = v_x D_y (= v_x D_x)$, the symbol D represents the Rhelogical operator,

Deflection w can be considered the product of two functions using the variable separation method
w(x,y,t)= W(x,y) T(t). (4)

2. ASSUMPTION REQUIRED

assuming that the plate's thickness l, varies in a circular fashion in one dimension, i.e.,

$$l = l_0\left[1 + B\left(1 - \sqrt{1 - \frac{x^2}{a^2}}\right)\right]. \quad (5)$$

Where B is a taper constant. We assume a one-dimensional linear fluctuation in density for non-homogeneity consideration, as

$$\rho = \rho_0\left(1 + m_1\frac{x}{a}\right). \quad (6)$$

Where the non- homogeneity constants is denoted by m_1 , $0 \le m_1 < 1$.

Consider a plate subjected to a continuous two-dimensional linear temperature distribution

$$\tau = \tau_0\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right). \quad (7)$$

E_1 and E_2 denote the Young's moduli along the x-and y-axes, respectively, evaluated at the reference temperature $\tau = 0$. The coefficient α characterizes the rate of change of the elastic modulus with respect to τ, this change in modulus can be expressed as:

$$E_x(\tau) = E_1\left[1 - A\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right],$$

$$E_y(\tau) = E_2\left[1 - A\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right],$$

$$G_{xy}(\tau) = G_0\left[1 - A\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right]. \quad (8)$$

Where the temperature gradient parameter is denoted by $A = \alpha\,\tau_0$ $(0 \le A < 1)$.

3. CLAMPED BOUNDARY CONDITION

The clamped boundary conditions are W = 0 and $\frac{\partial W}{\partial x}$ = 0, at x= 0, a; $\frac{\partial W}{\partial y} = 0$ at y = 0, a.

A two-term deflection function is assumed to satisfy the expression as:

$$W(x,y) = \left[\left(\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right)^2\right] * \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right]. \quad (9)$$

where A_1 and A_2 are arbitrary constants.

4. RAYLEIGH RITZ METHOD WITH RECTANGULAR PLATE

The Rayleigh Ritz method aims to ensure that the system's maximum potential (strain) energy is equals to its maximum kinetic energy, which can be succinctly stated as:

$$\delta(S - K) = 0. \quad (10)$$

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The strain energy and kinetic energy for plate vibration are expressed as:

$$S = \frac{1}{2}\int_0^a \int_0^b \begin{bmatrix} D_x\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_1 * \\ \left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right) + 4\, D_{xy}\left(\frac{\partial^2 W}{\partial x\, \partial y}\right)^2 \end{bmatrix} dxdy. \tag{11}$$

And $K = \frac{1}{2} P^2 \rho \int_0^a \int_0^b l\, W^2\, dxdy.$ (12)

The non-dimensional variables are introduce here to make our assumption simple and satisfied.

$X = \frac{x}{a}, \quad Y = \frac{y}{a},$

$E_1^* = \frac{E_1}{(1-v_x v_y)}, \; E_2^* = \frac{E_2}{(1-v_x v_y)}$ and

$E^* = v_x E_2^* = v_y E_1^*.$ (13)

5. SOLUTION OF FREQUENCY EQUATION

By using equation (13), along with (10), (11) and (12), we get

$$\delta(S-K) = \frac{Q}{2}\int_0^a \int_0^b \left[1 - A\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]\left[1 + B\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right]^3 \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_2^*}{E_1^*}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v_x \frac{E_2^*}{E_1^*}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{G_0}{E_1^*}(1-v_x v_y)\left(\frac{\partial^2 W}{\partial x\, \partial y}\right)^2\right] dxdy - d^2\int_0^a \int_0^b \left(1 + m_1\frac{x}{a}\right)\left[1 + B\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right] W^2 dxdy = 0. \tag{14}$$

The limits are now set as X varying between 0 to 1, and Y between 0 and $\frac{b}{a}$, respectively. By replacing the values of S & K in equation (14) with the help of equation (10), we obtain

$$(S_1^* - d^2 K_1^*) = 0. \tag{15}$$

Thus we have value for S_1^* and K_1^* define as:

$$S_1^* = \frac{Q}{2}\int_0^1 \int_0^{\frac{b}{a}} \left[1 - A(1-X)\left(1-\frac{Y\,a}{b}\right)\right] * \left[1 + B\left(1-\sqrt{1-X^2}\right)\right]^3 * \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_2^*}{E_1^*}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v_x \frac{E_2^*}{E_1^*}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{G_0}{E_1^*}(1 - v_x v_y)\left(\frac{\partial^2 W}{\partial x\, \partial y}\right)^2\right] dXdY. \tag{16}$$

And

$$K_1^* = \int_0^1 \int_0^{b/a} (1 + m_1 X)\left[1 + B\left(1-\sqrt{1-X^2}\right)\right] W^2 dXdY. \tag{17}$$

Where

$Q = \frac{1}{2}\frac{E_1\, l_0^{\,3}}{12(1-v_x v_y)}$ and $d^2 = \frac{12\, P^2 a^4 \rho(1-v_x v_y)}{E_1 l_0^{\,2}}.$

The unknown A_1 & A_2 in equation (15) result from the substitutions of W from equation (9). The following formula must be used to determine these two constants:

$\frac{\partial}{\partial A_n}[S_1^* - d^2 K_1^*] = 0,$ n=1, 2. (18)

Equation (18) can be simplified to provide the following form

$C_{q1}\, A_1 + C_{q2}\, A_2 = 0.$ (19)

The parametric constants and the frequency parameter are involved in C_{q1} & C_{q2} where q=1, 2. When the coefficients of equation (19) are determined to be non-zero, they must disappear. In this manner, the frequency equation was

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0. \tag{20}$$

By solving equation (20), one obtains a quadratic equation in d^2, yielding two roots. When $A_1 = 1$ is

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chosen to be substituted in equation (9), $A_2 = -\frac{c_{11}}{c_{12}}$ is obtained, and W becomes

$$W(x,y) = \left[\left(\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right)^2\right] * \left[1 + \left(-\frac{c_{11}}{c_{12}}\right)\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]. \quad (21)$$

Furthermore, it can be written as:

$$W(x,y) = \left[\left(XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right)^2\right] * \left[1 + \left(-\frac{c_{11}}{c_{12}}\right)XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right].$$

6. RESULT AND DISCUSSION

Duralumin' is a mixture of copper, magnesium, manganese and aluminium that we have used. To calculate the material properties, we use the thermal gradient (A), taper constant (B), non- homogeneity (m_1) and the aspect ratio (a/b) at different positions for the first two vibration modes, ($d_1$and d_2).

Our computation makes use of the following material parameters:

$\frac{E_2^*}{E_1^*} = 0.32$,

$\frac{E^*}{E_1^*} = 0.04$,

$\frac{G_0}{E_1^*} = 0.09$, E= 7.08× 10^10,

ρ_0 = 2.80× 10^3, G_0 = 2.632× 10^10, v = 0.345, a=3, 5 and b=2.

We have considered the orthotropic rectangular plate having circular variation in x-direction with thermal effect in which temperature is linear in both direction and density varies linear. Frequency values for the first two modes are computed using the Matlab software tool and the Rayleigh Ritz method. The results are presented and analyzed in the tables below. The length-to-breadth ratio of the plate is 1.5 and 2.5, respectively.

Table1, 2 and 3 illustrate the frequency for the initial two modes of thermal gradient (A), non homogeneity (m_1), and taper constant (B), respectively.

- In Table1 for thermal gradient (A) we take values (B=m_1=0.2, v=0.345, B=m_1=0.4, v=0.345, B=m_1=0.8, v=0.345).
- In Table2 for non homogeneity (m_1) we take values (A=B=0.2, v=0.345, A=B=0.4, v=0.345, A=B=0.8, v=0.345).
- In Table3 for taper parameter (B) we take values (A=m_1=0.2, v=0.345, A=m_1=0.4, v=0.345, A=m_1=0.8, v=0.345).
- In Table1 and Table2, both thermal gradient (A) and non homogeneity parameter (m_1), increases horizontally to the right at various points ranging from 0.2 to 0.8.

- Table3 now shows a thermal gradient (A) with non homogeneity (m_1) for the values (A=m_1=0.2, A=m_1=0.4 and A=m_1=0.8), as well as a taper parameter (B) ranging from 0.0 to 1.0.
- Tapering parameter (B) increases, so do the frequency mode values d_1 and d_2 .It is evident that when the frequency mode d_1 and d_2, listed in Table3, increase as taper parameter (B) is varied from 0.0 to 1.0, whereas in Table1 and Table2, the frequency mode values d_1 and d_2 decrease as thermal gradient and non-homogeneity rise within the range of 0.0 to 1.0.
- In Table3 taper parameter (B) has the highest values of frequency $d_1$and d_2 at A=m_1=0.2, at B =1.0.
- While in Table1 and Table2 has the highest values of frequency d_1 and d_2 in thermal gradient and non homogeneity respectively are B=m_1=0.8, at A=0.0 and A=B=0.8, at m_1=0.0.

Table4, 5 and 6 illustrate the frequency for the initial two modes for an aspect ratio of 2.5 relate to thermal gradient (A), non homogeneity (m_1) and taper constant (B), respectively.

- In Table4 for thermal gradient (A) we take values (B=m_1=0.2, v=0.345, B=m_1=0.4, v=0.345, B=m_1=0.8, v=0.345).
- In Table5 for non homogeneity (m_1) we take values (A=B=0.2, v=0.345, A=B=0.4, v=0.345, A=B=0.8, v=0.345).
- In Table6 for taper parameter (B) we take values (A=m_1=0.2, v=0.345, A=m_1=0.4, v=0.345, A=m_1=0.8, v=0.345).
- From Table4 thermal gradient (A), frequency mode values d_1 and d_2, decreases when we increase it from (0.0 to 0.8) at different stage (B=m_1=0.2,v=0.345, B=m_1=0.4,v=0.345, B=m_1=0.8, v=0.345). But values also horizontally decreased when we gradually increase the taper

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constant (B) and non homogeneity (m_1) from 0.2 to 0.8.

- From Table5 non homogeneity (m_1), frequency mode values d_1 and d_2, decreases vertically when we increase it from (0.0 to 1.0) at different stage (A=B=0.2, v=0.345, A=B=0.4, v=0.345, A=B=0.8, v=0.345). As thermal gradient (A) and taper constant (B) increase from 0.2 to 0.8, values show a horizontal increase.
- From Table6 taper constant (B), frequency mode values d_1 and d_2, increases vertically when we increase it from (0.0 to 1.0) at different stage (A= m_1=0.2, v=0.345, A= m_1=0.4, v=0.345, A= m_1=0.8, v=0.345). The mode values decrease along the horizontal direction as thermal gradient (A) and non homogeneity (m_1) increase between 0.2 to 0.8.
- All the three Tables 4, 5 and 6 shows the different behaviour for their mode values d_1 and d_2. Table4 mode values d_1 and d_2 decrease in both sides at different stages. Table5 mode values d_1 and d_2 decreases in vertically side and increases in horizontally side at different stages. Table6 mode values d_1 and d_2 increases in vertically side and decreases in horizontally at different stages.

Table-1. Thermal gradient (A) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

A	B=m_1=0.2,v=0.345		B=m_1=0.4, v=0.345		B= m_1=0.8,v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	46.2992	185.7404	46.4354	186.1372	47.2641	189.6615
0.2	45.1911	181.3104	45.3846	181.9717	46.3010	185.9494
0.4	44.0551	176.7695	44.3085	177.7087	45.3159	182.1620
0.6	42.8888	172.1088	43.2051	173.3410	44.3072	178.2946
0.8	41.6897	167.3184	42.0721	168.8606	43.2729	174.3419

Graph-1. Constant aspect ratio of 1.5, Thermal gradient (A) versus Frequency (d)

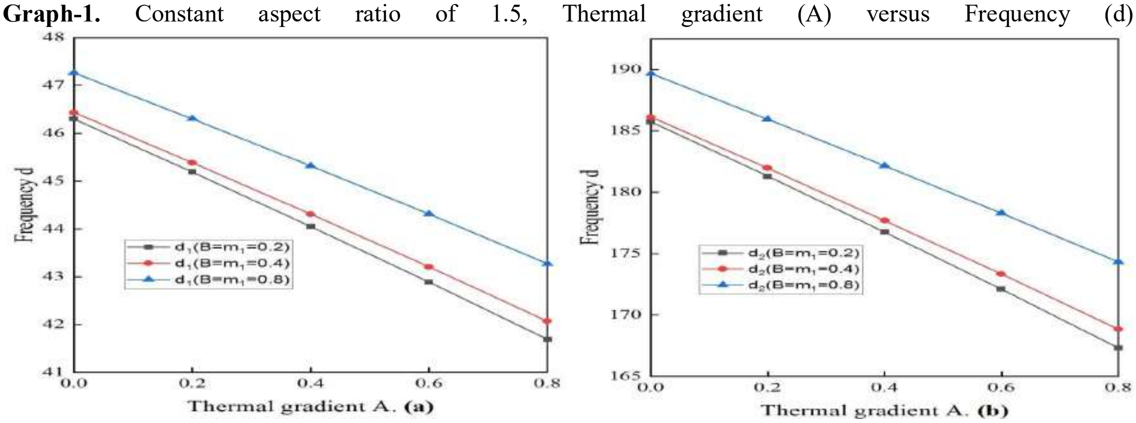

Table-2. Non-homogeneity (m_1) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

m_1	A=B=0.2, v=0.345		A=B=0.4, v=0.345		A=B=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	47.4079	190.2620	48.5773	195.0416	51.3319	207.559
0.2	45.1911	181.3104	46.2959	185.7713	48.9031	197.5105
0.4	43.2588	173.5141	44.3085	177.7087	46.7895	188.7928
0.6	41.5550	166.6440	42.5570	170.6123	44.9283	181.1364
0.8	40.0388	160.5303	40.9980	164.3034	43.2729	174.3419
1.0	38.6756	155.0437	39.5987	158.6464	41.7881	168.2590

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Graph-2. Constant aspect ratio of 1.5, Non-homogeneity constant (m_1) versus Frequency (d)

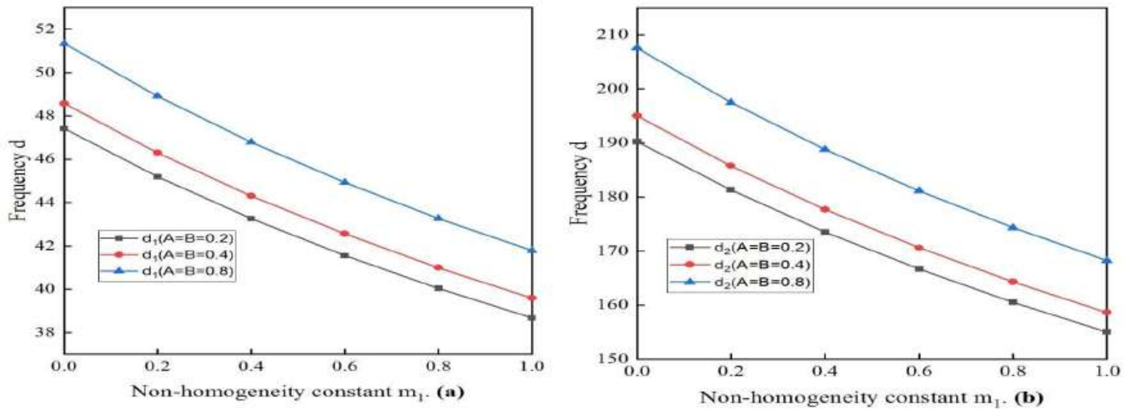


Table-3. Taper constant (B) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

B	A=m₁=0.2, v=0.345		A=m₁=0.4, v=0.345		A=m₁=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	43.1471	173.3386	40.2084	161.5327	35.0968	140.9973
0.2	45.1911	181.3104	42.1714	169.1683	36.9358	148.1418
0.4	47.4203	190.2276	44.3085	177.7087	38.9289	156.1225
0.6	49.8073	200.0321	46.5928	187.0947	41.0492	164.8782
0.8	52.3263	210.6572	48.9990	197.2598	43.2729	174.3419
1.0	54.9545	222.0329	51.5054	208.1345	45.5805	184.4455

Graph-3. Constant aspect ratio of 1.5, Tapering constant (B) versus Frequency (d)

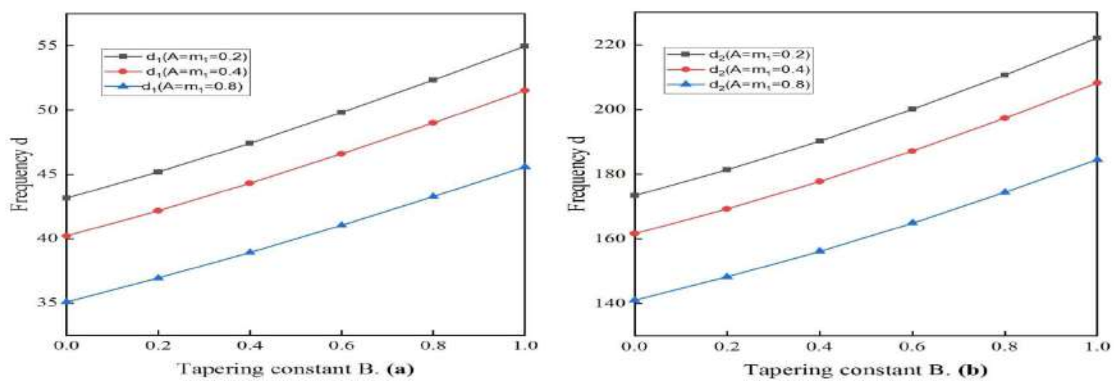


Table-4. Thermal gradient (A) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

A	B=m₁=0.2,v=0.345		B=m_1=0.4, v=0.345		B= m₁=0.8,v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	36.5347	147.6456	36.1889	145.7340	35.8758	143.4432
0.2	35.6333	143.9953	35.3190	142.2184	35.0569	140.1534
0.4	34.7085	140.2500	34.4272	138.6137	34.2183	136.7845
0.6	33.7583	136.4019	33.5115	134.9127	33.3584	133.3306
0.8	32.7806	132.4420	32.5701	131.1073	32.4755	129.7848

Graph-4. Constant aspect ratio of 2.5, Thermal gradient (A) versus Frequency (d)

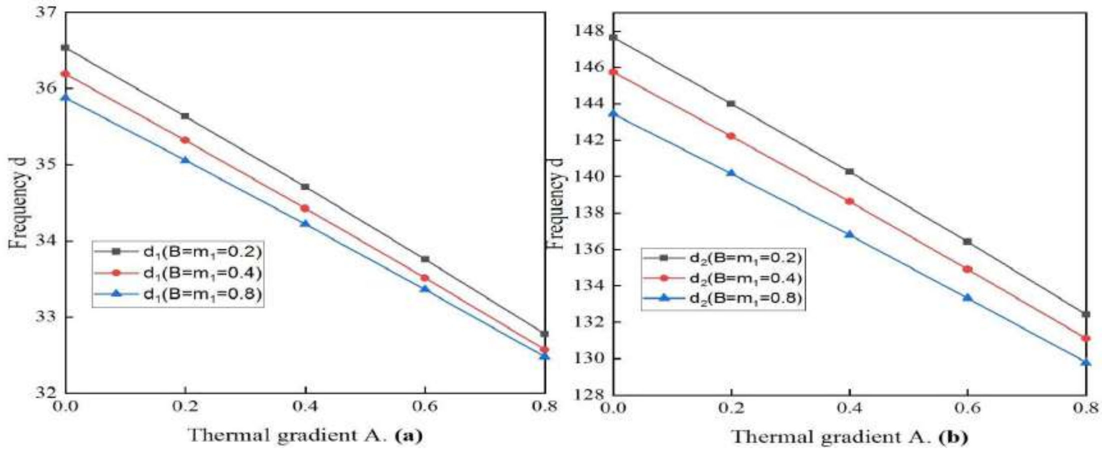


Table-5. Non-homogeneity (m_1) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

m_1	A=B=0.2, v=0.345		A=B=0.4, v=0.345		A=B=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	37.3812	151.1045	37.7439	152.1320	38.5252	154.4989
0.2	35.6333	143.9953	35.9713	144.9019	36.7018	147.0229
0.4	34.1098	137.8035	34.4272	138.6137	35.1151	140.5370
0.6	32.7664	132.3474	33.0663	133.0789	33.7180	134.8404
0.8	31.5701	127.4920	31.8551	128.1584	32.4755	129.7848
1.0	30.4961	123.1345	30.7680	123.7462	31.3611	125.2585

Graph-5. Constant aspect ratio of 2.5, Non homogeneity constant (m_1) versus Frequency (d)

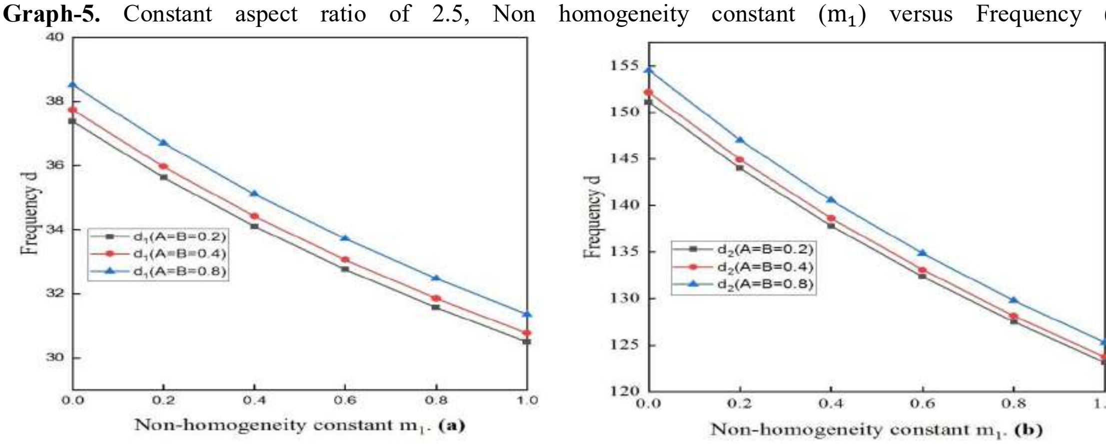
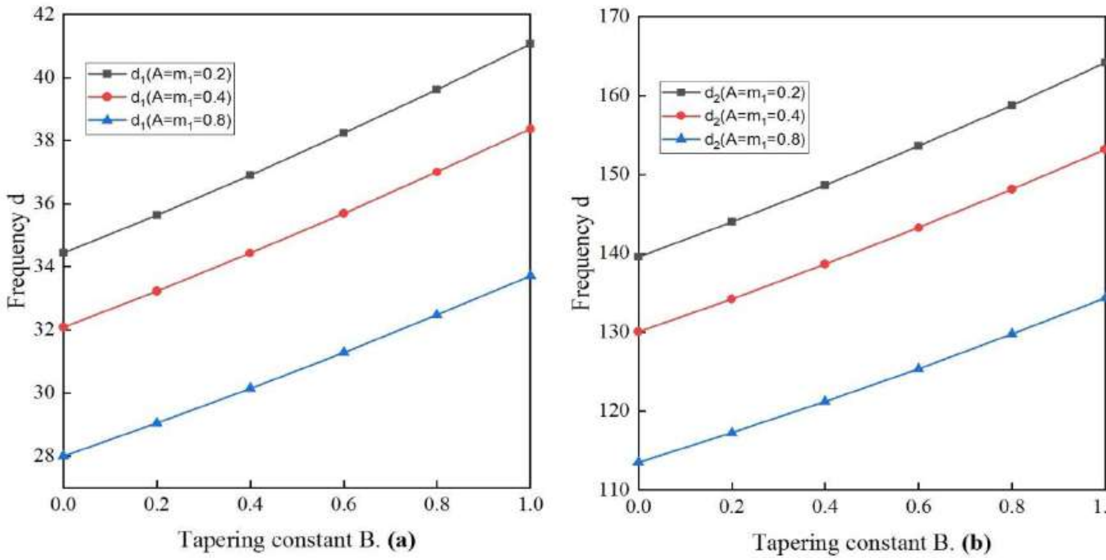


Table-6. Taper constant (B) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

B	A=m_1=0.2, v=0.345		A=m_1=0.4, v=0.345		A=m_1=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	34.4286	139.5874	32.0837	130.0803	28.0050	113.5434
0.2	35.6333	143.9953	33.2245	134.2192	29.0429	117.2628
0.4	36.9032	148.6702	34.4272	138.6137	30.1369	121.2181
0.6	38.2336	153.5990	35.6872	143.2506	31.2826	125.3966
0.8	39.6198	158.7678	36.9998	148.1163	32.4755	129.7848
1.0	41.0569	164.1620	38.3604	153.1964	33.7111	134.3691

Graph-6. Constant aspect ratio of 2.5, Tapering constant (B) versus Frequency (d)



CONCLUSION

This study presents the results of an investigation into the frequency distribution of orthotropic rectangular plates subjected to linear temperature changes and circular fluctuations in density and thickness. The rise in thermal (A), as shown in Table1, from 0.0 to 0.8, it increases horizontally and decrease vertically for different values of Taper constant (B) and non homogeneity (m_1) at $B=m_1=0.2$, $B=m_1=0.4$, $B=m_1=0.8$. However, Table2 and Table5 show a same behaviour for increase and decrease in frequency for temperature gradient (A) and non-homogeneity (m_1). Table3 and Table6 also behave in a similar manner for taper constant (B) from 0.0 to 1.0 at different values of $A=m_1=0.2$, $A=m_1=0.4$ and $A=m_1=0.8$. Table4 decrease vertically and horizontally for different mode values at $B=m_1=0.2$, $B=m_1=0.4$ and $B=m_1=0.8$, where thermal gradient (A) varies from 0.0 to 0.8. The frequency changes in modes d_1 and d_2, weather increasing or decreasing, happen very slowly due to the way the circular variation is implemented. Temperature variations, non-homogeneity and tapering significantly influence the vibration behaviour of plates. All mechanical and engineering structure can't be imagining without the role of these elements. Our research utilizes a machine framework to examine the impact of temperature material, non-homogeneity and tapering. A mathematical model with linear temperature and circular thickness can use in marine engineering, optical elements and latest applications. We need a design structure which can balanced the natural frequency and make a suitable mode variation, so our plate material becomes less weight and reduce the unwanted vibration. Hence in a good mechanical structure you need to know the effect of temperature, homogeneity and tapering in a plate material, and our study will helps that process.

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