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Study the Free Vibration of Square Plate with Bi-Linear Circular Varying Thickness and Varying Bi-Parabola Thermal Effect

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Abstract- This paper presents an analytical study on the free vibration characteristics of a visco-elastic square plate with bi-dimensional thickness variation under prescribed thermal conditions. The plate is considered fully clamped along all four edges, and its thickness is assumed to vary linearly along both in-plane directions. Using classical plate theory, the governing vibration equation is established, and the Rayleigh–Ritz method is employed to obtain an approximate frequency equation. The first two natural frequencies are computed for different values of the taper parameters while keeping the thermal gradient fixed. The numerical results indicate that an increase in taper parameters significantly enhances the natural frequencies for both modes of vibration due to increased stiffness resulting from thickness variation. The influence of taper parameters becomes more pronounced for higher vibration modes. The computed frequencies are presented through tables and combined graphical representations, highlighting the role of geometric non-uniformity in controlling the dynamic behavior of visco-elastic plate structures.

Keywords- Square plate; Taper parameter; Variable thickness; Free vibration; Rayleigh–Ritz method.

1. INTRODUCTION

Structural components with spatially varying geometry are increasingly used in modern engineering applications to meet demands for high strength, reduced weight, and improved dynamic performance. Among these components, plates with variable thickness play a critical role in aerospace systems, mechanical assemblies, and civil infrastructure, where geometric tailoring is often employed to optimize stiffness and vibration characteristics.

Visco-elastic materials are widely adopted in such systems due to their inherent energy dissipation capability and improved vibration control. When plate thickness varies along one or more directions, the resulting stiffness non-uniformity leads to marked changes in the natural frequencies of the structure. In practical situations, these plates may also operate under thermal environments; however, the geometric influence often governs the overall vibration response when thermal conditions are prescribed or moderately varying.

Existing literature includes several analytical and numerical studies on vibration behavior of plates with uniform and non-uniform thickness, as well as investigations employing approximate methods such as the Rayleigh–Ritz technique. Nevertheless, a detailed parametric study focusing on the effect of taper parameters in visco-elastic square plates with bi-dimensional thickness variation, particularly under fully clamped boundary conditions, is still limited.

In this study, the free vibration characteristics of a visco-elastic square plate with thickness varying linearly in both directions are investigated with emphasis on taper parameters. Using the Rayleigh–Ritz method, an approximate frequency equation is derived, and the first two natural frequencies are evaluated for different combinations of taper parameters under prescribed thermal conditions. The numerical results, presented in tabular and graphical form, clearly demonstrate the influence of thickness variation on the vibrational behavior of the plate.

2. MATHEMATICAL FORMULATION

A visco-elastic square plate of side length a with bi-dimensional thickness variation is considered for the free vibration analysis. The plate is assumed to be isotropic and fully clamped along all four edges. The

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formulation is developed to study the influence of taper parameters on the vibration behaviour of the plate under prescribed thermal conditions.

Based on classical plate theory, the governing differential equation for transverse vibration of a visco-elastic plate with spatially varying flexural rigidity D_1 is expressed as:

$[D_1(W_{xxxx} + 2W_{xx}yy + Wyyyy) + D_{1,x}(W_{xxx} + W_{x}yy) + D_{1,\gamma}(Wyyy + W_{xx}y) + D_{1,xx}W_{xx} + D_{1,\gamma\gamma}W_{\gamma\gamma} + 2(1 - \nu)D_{1,x\gamma}W_{x\gamma}] + \nu hpW = 0$

The flexural rigidity of the plate is defined as:

$D_1 = Eh^3 / [12(1 - \nu^2)]$

The plate thickness is assumed to vary linearly along both in-plane directions and is given by:

$h = h_0(1 + \beta_1 x/a)(1 + \beta_2 y/a)$

A prescribed thermal field is considered, and the Young's modulus is expressed as a temperature-dependent quantity as:

$E = E_0[1 - \alpha(1 - x/a)(1 - y/a)]$

Substituting the expressions for elastic modulus and thickness into the rigidity relation, the modified flexural rigidity is obtained as:

$D_1 = [E_0h_0^3(1 - \alpha(1 - x/a)(1 - y/a))(1 + \beta_1 x/a)^3(1 + \beta_2 y/a)^3] / [12(1 - \nu^2)]$

To satisfy the fully clamped boundary conditions, a two-term admissible deflection function is assumed. The Rayleigh–Ritz method is employed to derive the frequency equation by minimizing the functional:

$\delta(V^* - T^*) = 0$

The resulting formulation leads to a characteristic equation in terms of the frequency parameter, from which the first two natural frequencies are obtained. In this paper, the formulation is primarily used to investigate the effect of taper parameters β_1 and β_2 on the vibration characteristics of the plate for selected values of the thermal gradient parameter α.

3. RESULTS AND DISCUSSION

This section presents and discusses the numerical results obtained to study the influence of taper parameters on the free vibration behavior of a visco-elastic square plate. The first two natural frequencies are computed for varying values of the taper parameters β1 and β2, while the thermal gradient parameter α is maintained at prescribed values. The computed frequencies are presented in tables and combined plots to illustrate the effect of bi-dimensional thickness variation on the vibration characteristics of the plate under clamped boundary conditions.

3.2 EFFECT OF TAPER PARAMETER β1

To investigate the influence of the taper parameter on the vibration characteristics of the visco-elastic square plate, the first two natural frequencies are evaluated for increasing values of the taper parameter β1, while the thermal gradient parameter α and the taper parameter β2 are kept constant. The analysis is performed for three different values of the thermal gradient, namely α=0.0, 0.40.40.4, and 0.80.80.8, under fully clamped boundary conditions.

The computed values of the natural frequencies for varying β1 and prescribed values of the thermal gradient are presented in Table 1,2,3 and the corresponding graphical representation is shown in Fig. 1.

Table 1. Variation of natural frequency with thermal gradient α for different values of taper parameters β1=β2 under fully clamped boundary conditions.

α	$\beta_1=\beta_2=0$		$\beta_1=\beta_2=0.4$		$\beta_1=\beta_2=0.8$	
0	35.99	10.88	47.41	185.60	61.60	242.74
0.2	34.31	134.28	45.58	178.76	59.53	235.54
0.4	32.52	127.34	43.66	171.64	57.34	228.13
0.6	30.63	119.99	41.61	164.23	55.01	220.47
0.8	28.60	112.18	39.42	156.47	52.51	212.56
1.0	26.39	103.78	37.03	148.32	49.81	204.36

Table 2. Variation of natural frequency with taper parameter β1 for different values of thermal gradient α with taper parameter β2 kept constant.

β_1	$\alpha=\beta_2=0$		$\alpha=\beta_2=0.4$		$\alpha=\beta_2=0.8$	
0	35.99	140.88	39.50	154.60	43.40	170.64
0.2	38.58	150.94	42.43	166.12	46.68	183.80
0.4	41.35	161.78	45.54	178.48	50.15	197.88
0.6	44.26	173.29	48.80	191.57	53.77	212.72
0.8	47.28	185.36	52.17	205.27	57.49	228.22
1.0	50.39	197.90	55.64	219.48	61.32	244.27

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Table 3. Variation of natural frequency with taper parameter β2 for different values of thermal gradient α with taper parameter β1 kept constant.

β_2	$\alpha=\beta_1=0$		$\alpha=\beta_1=0.4$		$\alpha=\beta_1=0.8$	
0	35.99	140.88	39.41	158.33	43.87	184.92
0.2	38.74	148.52	42.32	170.92	47.00	197.40
0.4	40.71	159.07	45.22	182.69	50.30	214.04
0.6	43.85	166.31	47.34	195.22	53.95	226.19
0.8	46.12	175.07	51.17	209.53	57.62	242.50
1.0	49.44	186.24	54.39	223.40	61.56	268.79

The combined influence of thermal gradient and taper parameters on the natural frequencies is illustrated graphically in Fig. 1 using the numerical results from Tables 1–3.

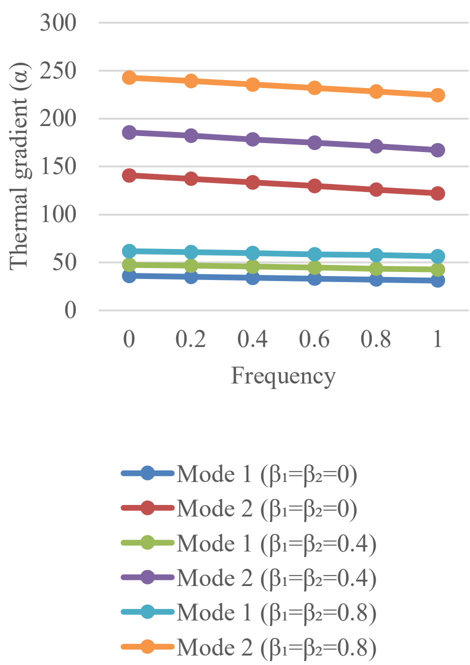

Fig 1: Variation of natural frequency with thermal gradient for different values of foundation parameters.

The results show that, for a fixed thermal gradient, the natural frequency of both vibration modes increases as the taper parameter β1 increases from 0.0 to 1.0 for the specified values of α and β2. This increase is observed consistently for both modes, indicating that higher taper stiffness enhances the vibrational frequency of the plate.

4. CONCLUSION

This study presents an analysis of the free vibration characteristics of a visco-elastic square plate with bi-directional thickness variation under prescribed thermal conditions. The results demonstrate that the natural frequencies of both vibration modes increase consistently with increasing taper parameters β1 and β2 due to the associated enhancement in plate stiffness. The influence of taper parameters is found to be more pronounced for higher vibration modes. The study confirms the significant role of thickness variation in controlling the dynamic response of visco-elastic plate structures.

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