To Study the Free Vibration of Square Plate with Bi-Linear Circular Varying Thickness and Thermal Effect

Manju Bala[1], Ashish Kumar Sharma[2*]

[1]*Research Scholar, Department of Mathematics, Arni University, Kangra (H.P.)*
[*2]*Associate Professor, Department of Mathematics, Arni University, Kangra (H.P.)*

Abstract- The free vibration behavior of a visco-elastic square plate with bi-dimensional thickness variation subjected to thermal gradients is investigated in the present study. The plate is assumed to be fully clamped along all edges, and a temperature-dependent modulus of elasticity is considered to account for thermal effects. The governing vibration equation is formulated using classical plate theory, and an approximate frequency equation is derived by applying the Rayleigh–Ritz method with a two-term admissible deflection function. The first two natural frequencies are evaluated for different values of the thermal gradient parameter while considering selected taper parameters for thickness variation. The numerical results show that the natural frequencies decrease consistently with increasing thermal gradient due to thermal softening of the plate material. However, for a fixed thermal gradient, higher taper parameters lead to increased frequencies owing to enhanced structural stiffness. The results, presented in tabular and graphical form, provide useful insight into the dynamic response of visco-elastic plates operating under non-uniform thermal environments.

Keywords- Square plate; Thermal gradient; Free vibration; Variable thickness; Rayleigh–Ritz method.

1. INTRODUCTION

Plate elements made of visco-elastic materials are integral to many engineering systems operating under elevated temperature conditions, such as aerospace structures, nuclear installations, power generation equipment, and marine components. In such environments, temperature variations significantly influence material stiffness, which in turn alters the dynamic response of structural elements. Understanding the effect of thermal gradients on vibration behavior is therefore essential for ensuring structural reliability and performance.In recent engineering designs, variable-thickness plates have gained considerable attention due to their ability to provide improved stiffness distribution without excessive increase in structural weight.

When these geometric variations are combined with temperature-dependent material properties, the vibration characteristics deviate substantially from those predicted by classical uniform-thickness plate theories. This effect is especially pronounced in visco-elastic materials, where the elastic modulus decreases with increasing temperature.

Although extensive research has been carried out on vibration analysis of plates under thermal loading and on plates with non-uniform thickness, studies addressing the combined influence of thermal gradients and bi-dimensional thickness variation in visco-elastic square plates are relatively scarce. Moreover, investigations considering fully clamped boundary conditions, which are common in practical applications, remain limited.

The present work examines the free vibration behavior of a visco-elastic square plate with thickness varying linearly along both in-plane directions and subjected to a two-dimensional thermal gradient.

A temperature-dependent modulus of elasticity is incorporated into the formulation, and the Rayleigh–Ritz method is employed to obtain approximate frequency equations. The effect of thermal gradient on the first two natural frequencies is analyzed for different taper parameters, and the resulting trends are discussed using numerical and graphical representations.

2. MATHEMATICAL FORMULATION

A visco-elastic square plate of side length a with bi-dimensional thickness variation is considered for free vibration analysis.

The plate is assumed to be isotropic and fully clamped along all four edges. The effect of thermal gradient and taper parameters on the vibration behaviour is incorporated through spatially varying material properties and plate thickness.

Based on classical plate theory, the governing differential equation for transverse vibration of a visco-elastic plate with variable flexural rigidity D_1 is expressed as:

$[D_1(W_{xxxx} + 2W_{xxyy} + W_{yyyy}) + D_{1,x}(W_{xxx} + W_{xyy}) + D_{1,\gamma}(W_{yyy} + W_{xxy}) + D_{1,xx}W_{xx} + D_{1,\gamma\gamma}W_{\gamma\gamma} + 2(1 - v)D_{1,x\gamma}W_{x\gamma}] + vhpW = 0$

The flexural rigidity of the plate is given by:

$D_1 = Eh^3 / [12(1 - v^2)]$

A steady-state two-dimensional temperature distribution across the plate is assumed as:

$\tau = \tau_0 (1 - x/a) (1 - y/a)$

The temperature-dependent modulus of elasticity is approximated by:

$E = E_0 (1 - \gamma\tau)$

Substituting the temperature distribution, the elastic modulus becomes:

$E = E_0 [1 - \alpha (1 - x/a) (1 - y/a)]$, where $\alpha = \gamma\tau_0$

The plate thickness is assumed to vary linearly along both in-plane directions and is expressed as:

$h = h_0 (1 + \beta_1 x/a) (1 + \beta_2 y/a)$

Substituting the expressions for elastic modulus and thickness into the rigidity relation, the modified flexural rigidity is obtained as:

$D_1 = [E_0 h_0^3 (1 - \alpha (1 - x/a) (1 - y/a))(1 + \beta_1 x/a)^3(1 + \beta_2 y/a)^3] / [12(1 - v^2)]$

To satisfy the fully clamped boundary conditions, a two-term admissible deflection function is assumed. The Rayleigh–Ritz method is employed to derive the frequency equation by minimizing the functional:

$\Delta (V^* - T^*) = 0$

The resulting formulation leads to a characteristic equation in terms of the frequency parameter, from which the first two natural frequencies are obtained for different values of the thermal gradient parameter α and taper parameters β_1 and β_2.

3. RESULTS AND DISCUSSION

In this section, the numerical results obtained from the Rayleigh–Ritz formulation are presented and discussed to examine the effect of thermal gradient on the vibration characteristics of the visco-elastic square plate.

The first two natural frequencies are evaluated for different values of the thermal gradient parameter α, while the taper parameters β_1 and β_2 are kept at selected fixed values.

The results are summarized in tabular form and illustrated graphically to highlight the influence of thermal loading on the dynamic response of the plate under fully clamped boundary conditions.

3.1 EFFECT OF THERMAL GRADIENT α

To examine the influence of thermal gradient on the vibration characteristics of the visco-elastic square plate, the first two natural frequencies are evaluated for increasing values of the thermal gradient parameter α, while the taper parameters are kept constant.

The analysis is carried out for three different cases of taper parameters, namely $\beta_1 = \beta_2 = 0.0$, 0.40.40.4, and 0.80.80.8, under fully clamped boundary conditions.

The computed values of the natural frequencies corresponding to different values of the thermal gradient and taper parameters are presented in Tables 1–3.

Table 1. Variation of natural frequency with thermal gradient α for different taper parameters $\beta_1 = \beta_2$.

α	$\beta_1=\beta_2=0$		$\beta_1=\beta_2=0.4$		$\beta_1=\beta_2=0.8$	
0	35.99	140.88	47.41	185.60	61.60	242.74
0.2	35.08	137.31	46.49	182.08	60.61	239.19
0.4	34.15	133.65	45.54	178.48	59.60	235.59
0.6	33.19	129.88	44.58	174.81	58.56	231.93
0.8	32.19	126.01	43.58	171.07	57.49	228.22
1.0	31.17	122.00	42.57	167.24	56.40	224.44

Table 2. Variation of natural frequency with taper parameter β1 for different values of thermal gradient α with β2 fixed.

β₁	α=β₂=0		α=β₂=0.4		α=β₂=0.8	
0	35.99	140.88	39.50	154.60	43.40	170.64
0.2	38.58	150.94	42.43	166.12	46.68	183.80
0.4	41.35	161.78	45.54	178.48	50.15	197.88
0.6	44.26	173.29	48.80	191.57	53.77	212.72
0.8	47.28	185.36	52.17	205.27	57.49	228.22
1.0	50.39	197.90	55.64	219.48	61.32	244.27

Table 3. Variation of natural frequency with taper parameter β2 for different values of thermal gradient α with β1 fixed.

β₂	α=β₁=0		α=β₁=0.4		α=β₁=0.8	
0	35.99	140.88	39.41	158.33	43.87	184.92
0.2	38.74	148.52	42.32	170.92	47.00	197.40
0.4	40.71	159.07	45.22	182.69	50.30	214.04
0.6	43.85	166.31	47.34	195.22	53.95	226.19
0.8	46.12	175.07	51.17	209.53	57.62	242.50
1.0	49.44	186.24	54.39	223.40	61.56	268.79

The variation of natural frequency with thermal gradient for different taper parameters is illustrated graphically in The results indicate that the natural frequency of both vibration modes decreases with increasing thermal gradient α for all values of β1 and β2.

In contrast, for fixed thermal conditions, the frequency increases steadily with increasing taper parameter β1 and foundation parameter β2, reflecting enhanced structural stiffness.

This trend is consistent for both modes of vibration.

4. CONCLUSION

The free vibration behavior of a visco-elastic square plate with bi-dimensional thickness variation subjected to thermal gradients has been investigated using the Rayleigh–Ritz method. The results show that the natural frequencies of both vibration modes decrease with an increase in the thermal gradient parameter due to thermal softening of the plate material. It is also observed that higher taper parameters partially offset this reduction by enhancing the structural stiffness. The findings provide useful insight for the design of plate structures operating under non-uniform thermal environments.

Reference

[1] Leissa, A.W., *Vibration of Plates*. NASA SP-160, U.S. Government Printing Office, Washington DC, 1969.

[2] Timoshenko, S.P., Woinowsky-Krieger, S., *Theory of Plates and Shells*. McGraw-Hill, New York, 1959.

[3] Nayak, A.K., Bandyopadhyay, J.N., "Thermal effects on free vibration of non-homogeneous plates," *Journal of Sound and Vibration*, Vol. 121, No. 2, pp. 307–317, 1988.

[4] Batra, R.C., "Thermal effects on vibration of viscoelastic plates," *Journal of Thermal Stresses*, Vol. 10, pp. 123–138, 1987.

[5] Shukla, K.K., "Free vibration of non-homogeneous rectangular plates under thermal gradient," *Journal of Sound and Vibration*, Vol. 244, No. 3, pp. 475–491, 2001.

[6] Lal, R., "Thermal behavior of visco-elastic plates with variable thickness," *Applied Mathematical Modelling*, Vol. 26, pp. 395–405, 2002.

[7] Reddy, J.N., *Energy and Variational Methods in Applied Mechanics*. Wiley, New York, 1984.

[8] Zhou, D., Cheung, Y.K., "Vibration of plates with temperature-dependent elastic moduli," *International Journal of Mechanical Sciences*, Vol. 43, pp. 657–682, 2001.