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VIBRATION ANALYSIS OF NON-HOMOGENEOUS ISOTROPIC RECTANGULAR PLATE WITH NON-UNIFORM CIRCULARLY VARYING THICKNESS AND LINEARLY VARYING TEMPERATURE DISTRIBUTION

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ABSTRACT

This paper investigates the influence of structural characteristics on the vibration behaviour of a non-homogeneous rectangular plate with SSSS (simply supported on all sides) boundary conditions. The temperature variation across the plate is assumed to be linear, while the thickness variation is considered circular along the x-direction. The Rayleigh-Ritz method is employed to solve the differential equations for the first two vibration modes across different values of the structural parameter. The numerical results are shown in both tabular and graphical representations, and the plate is composed of an isotropic, visco-elastic material. MATLAB software is utilized to compute various parameters of the plate, including the thermal gradient, non-homogeneity, and taper characteristics.

KEY WORDS: vibration, simply supported, non-homogeneity, isotropic, aspect ratio, taper parameter, thermal gradient, flexural rigidity, kinetic energy, strain energy.

INTRODUCTION

Vibration analysis is the process of studying the oscillation or fluctuations of mechanical structure or its components. It is basically used to monitor the health of machinery, detect issues early and improve efficiency. Vibration analysis plays vital role in the field of engineering design. Every structure or component has its natural frequency and if the operating frequency of the system matches this natural frequency, it can lead to resonance which causes excessive vibrations. All engineering machines and structures having vibration and we cannot ignore the effect of vibration. Vibration may be detrimental in structural engineering, including earthquake engineering. They could lead to the failure of the structure. To get around this, a mechanical engineer must first understand the first few modes of vibration before coming up with a design. The materials are being developed to provide greater strength and efficiency based on requirements, durability, and reliability. The main objective of vibration research is to reduce unnecessary and unmanageable vibration by designing buildings and machines with accuracy and appropriateness.

A comprehensive study of the vibration characteristics of plates, covering theoretical formulations, analytical solutions, and practical applications, including rectangular, circular, and annular plates, under different boundary conditions [1]. Rayleigh Ritz method is used to solve the differential equation with linear variation in density and circular variation in Poisson's ratio on time period of vibration of rectangular plate [2]. A nonlinear vibrations of viscoelastic rectangular plates were investigated by using Von Karman nonlinear strain-displacement relationship [3]. The effect of thermal gradient on vibration of square plate of varying thickness is studied [4]. Effect of plates parameter on vibration of isotopic tapered rectangular plate has been studied under thermal condition [5]. Plate parameters study on tapered non-homogeneous rectangular plate with different boundary condition are investigated [6]. In non-uniform visco-elastic rectangular plate deflection for first two modes of vibration is calculated for different values of structural parameters [7]. A study of characteristics of mechanical vibration of plates with variable thickness and calculation are carried out with the help of Matrix laboratory computer software [8]. A high order triangular plate bending element is

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used in isotropic rectangular plates with linearly varying thickness in one direction [9]. The study of frequency and buckling of flexural vibration of isotropic and orthotropic rectangular plates using Rayleigh method [10]. An approximate formula for transverse vibration frequencies of rectangular plates with various boundary conditions using the Rayleigh method and validates it against accurate analyses [11]. The wave propagation approach and Hamilton's principle to derive natural frequencies of isotropic plates under various boundary conditions, validated through MATLAB simulations [12]. An exact solution for the free vibration of simply supported transversely isotropic thick plates using displacement potential functions, validated against analytical and finite element results for various thickness and aspect ratios [13].The impact of edge-boundary conditions on the free vibrations of isotropic and cross-ply laminated plates using a three-dimensional elasticity-based formulation, validated against exact and numerical results [14]. The study conducts static analysis of isotropic rectangular plates with various boundary conditions and loads using MATLAB-based finite element analysis, showing close agreement with classical exact solutions [15]. The Rayleigh–Ritz method with specially formulated admissible functions to analyze the vibration characteristics of rectangular plates with elastic edge supports, achieving high accuracy and convergence [16]. This study analyses the vibrational frequencies of non-homogeneous parallelogram plates with circular thickness variations using the Rayleigh–Ritz method, incorporating effects of temperature, skew angle, and material non-homogeneity [17]. The effect of linear thickness variations on the vibration of a viscoelastic rectangular plate with clamped boundary conditions is analysed. Using the Rayleigh–Ritz technique, frequency equations, logarithmic decrement, time period, and deflection for different modes are determined[18].

The deflection function is used to determine the modes of frequency for different values of thermal gradient, taper constant, and non-homogeneity, as well as employing aspect ratios of 1.5 and 2.5. The natural vibration of an isotropic non-homogeneous rectangular plate with a one-dimensional circularly variable thickness and density parameter under SSSS boundary conditions under the distribution of a two-dimensional temperature field is studied using the Rayleigh-Ritz technique.

In the present investigation solution of first two mode of vibration obtained in rectangular plate made of duralumin material with circular variation of the thickness in one direction. Assuming that the plate is simply supported on all four edges, the numerical values of the first two modes of the frequency at various structural parameter values for the SSSS boundary condition are shown in a tabular and graph format.

The differential equation of motion of visco-elastic isotropic plate may be written as [5]

$$\frac{\partial^2\tau_x}{\partial x^2} + 2\frac{\partial^2\tau_{xy}}{\partial x\,\partial y} + \frac{\partial^2\tau_y}{\partial y^2} = \rho h\frac{\partial^2\xi}{\partial t^2} \qquad (1)$$

where x and y represent the plate geometry's coordinates, τ_x and τ_y represent the bending moments, τ_{xy} represents the twisting moment per unit plate length, ρ represents the mass per unit volume, h represents the plate's thickness, and ξ represents the displacement at time t.

The expressions for τ_x, τ_y and τ_{xy} are given by [18]

$$\tau_x = -\tilde{D}D_1\left[\frac{\partial^2\xi}{\partial x^2} + \nu\frac{\partial^2\xi}{\partial y^2}\right],$$

$$\tau_y = -\tilde{D}D_1\left[\frac{\partial^2\xi}{\partial x^2} + \nu\frac{\partial^2\xi}{\partial y^2}\right] \text{ and}$$

$$\tau_{xy} = -\tilde{D}D_1(1-\nu)\frac{\partial^2\xi}{\partial x\,\partial y} \qquad (2)$$

where $\tilde{D}$ is the representation for visco-elastic operator. In this case, D_1 represents the material's flexural rigidity and is written as[2]

$$D_1 = \frac{Eh^3}{12(1-\nu^2)} \qquad (3)$$

Deflection ξ can be considered the product of two functions using the variable separation method[6]

$$\xi(x, y, t) = \phi\,(x, y) \cdot T(t) \qquad (4)$$

ASSUMPTION REQUIRED

one dimensional circular variation in thickness as [2]

$$h = h_0\left[1 + \beta\left(1 - \sqrt{1 - \frac{x^2}{a^2}}\right)\right] \qquad (5)$$

where $\beta, (0 \le \beta \le 1)$ is known as tapering parameter and thickness of plate becomes constant at $x = 0$ and for non-homogeneity (ρ) consideration, assumed one dimensional circular variation in Poisson's ratio (ν) as

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$$\rho = \rho_0\left[1 + m_2\frac{x}{a}\right], \tag{6}$$

$$\nu = \nu_0\left[1 - m_1\left(1 - \sqrt{1 - \frac{x^2}{a^2}}\right)\right] \tag{7}$$

where $m_2, (0 \le m_2 \le 1)$ and m_1 $(0 \le m_1 < 1)$are known as non-homogeneity constant corresponding to density and Poisson's ratio.

The plate is subjected to steady two-dimensional linear temperature distributions as[3]

$$\bar{\tau} = \bar{\tau}_0\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right) \tag{8}$$

Therefore the temperature dependent modulus of elasticity is taken as[5]

$$E(\tau) = E_0(1- \gamma\ \bar{\tau}) \tag{9}$$

$$E(\tau) = E_0\left[1- \gamma\ \bar{\tau}_0\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right]$$

$$E(\tau) = E_0\left[1- \alpha\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right] \tag{10}$$

Where $\alpha = \gamma\bar{\tau}_0$, $(0 \le \alpha < 1)$

BOUNDARY CONDITION

For SSSS , the boundary conditions are [6]

$$\phi = \frac{\partial^2\phi}{\partial x^2} = 0 \ \text{ at } \ x = 0, a$$

and $\phi = \frac{\partial^2\phi}{\partial y^2} = 0$ at $y = 0, b$

The deflection function (i.e. maximum displacement) which satisfy boundary condition given in as:

$$\phi\,(x, y) = \left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\left[A_1 + \right.$$
$$\left. + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right] \tag{11}$$

where A_1 and A_2 are arbitrary constants.

RAYLEIGH RITZ METHOD IN RECTANGULAR PLATE

We are using Rayleigh Ritz technique (i.e., maximum strain energy S_E must equal to maximum kinetic energy K_E) in order to obtain frequency equation and time period for both modes of vibrations. As a result, we need to have:

$$\delta(S_E - K_E) = 0 \tag{12}$$

The K_E and S_E formula are provided by [7]

$$K_E = \frac{1}{2}P^2\int_0^a \int_0^b \rho h\phi^{\,2}dxdy \tag{13}$$

$$S_E = \frac{1}{2}\int_0^a \int_0^b D_1 \times \left[\left(\frac{\partial^2\phi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\phi}{\partial y^2}\right)^2 + \right.$$
$$\left. + 2\nu\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2\phi}{\partial x\,\partial y}\right)^2\right]dxdy \tag{14}$$

The non-dimensional variables are outlined to make the computation simple and convenient:

$$X = \frac{x}{a},\quad Y = \frac{y}{b},\quad \bar{h} = \frac{h}{a} \quad \text{and} \quad \bar{\phi} = \frac{\phi}{a} \tag{15}$$

SOLUTION OF FREQUENCY EQUATION

on using the above assumptions along with (15) ; equation (13) and (14) becomes

$$K_E = \frac{1}{2}p^2\int_0^a\int_0^b \rho_0\left[1 + m_2\frac{x}{a}\right]h_0\left[1 + \beta\left(1 - \sqrt{1 - \frac{x^2}{a^2}}\right)\right]\phi^2 dydx$$

$$= \frac{1}{2}\rho_0 p^2 h_0\int_0^a\int_0^b\left[1 + m_2\frac{x}{a}\right]\left[1 + \beta\left(1 - \sqrt{1 - \frac{x^2}{a^2}}\right)\right]\phi^2 dydx$$

Substituting $X = \frac{x}{a}$ | $Y = \frac{y}{a}$

$x \to 0 \Rightarrow X \to 0$
$x \to a \Rightarrow X \to 1$

$y \to 0 \Rightarrow Y \to 0$
$y \to b \Rightarrow Y \to \frac{b}{a}$

$$K_E = \frac{1}{2}p_0 p^2\overline{h_0}\,a^5\int_0^1\int_0^{b/a}[1 + m_2X]\left[1 + \right.$$
$$\left. + \beta\left(1 - \sqrt{1 - X^2}\right)\right]\bar{\phi}^2 dXdY \tag{16}$$

And

$$S_E = \frac{1}{2}\int_0^a\int_0^b D_1\left[\left(\frac{\partial^2\phi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\phi}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2\phi}{\partial x\,\partial y}\right)^2\right]dxdy$$

$$= \frac{E\bar{h}_0^3 a^3}{24(1-\nu^2)}\int_0^1\int_0^{b/a}\left[1 - \alpha(1 - X)(1 - Y(a/\right.$$
$$b)]\left[1 + \quad \beta\left(1 - \sqrt{1 - X^2}\right)\right]^3\left[\left(\frac{\partial^2\bar{\phi}}{\partial X^2}\right)^2 + \right.$$

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$$\left(\frac{\partial^2\bar{\phi}}{\partial y^2}\right)^2 + \quad 2\nu\frac{\partial^2\bar{\phi}}{\partial x^2}\frac{\partial^2\bar{\phi}}{\partial y^2} + 2(1 - \nu)\left(\frac{\partial^2\bar{\phi}}{\partial x\,\partial y}\right)^2\Bigg]\,dXdY \qquad (17)$$

Using equation (15) and (16) in equation (12) represents the necessary frequency parameter.

$$\delta(S_E^* - \lambda^2 K_E^*) = 0 \qquad (18)$$

$$S_E^* = \int_0^1\int_0^{b/a}\left[1 - \alpha(1 - X)(1 - Y(a/b))\right]\Bigg[1 + \beta\left(1 - \sqrt{1 - X^2}\right]^3\left(\frac{\partial^2\bar{\phi}}{\partial x^2}\right)^2 + \left(\frac{\partial^2\bar{\phi}}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2\bar{\phi}}{\partial x^2}\frac{\partial^2\bar{\phi}}{\partial y^2} + 2(1 - \nu)\left(\frac{\partial^2\bar{\phi}}{\partial x\,\partial y}\right)^2\Bigg]\,dXdY \qquad (19)$$

$$k_E^* = \int_0^1\int_0^{b/a}\left[1 + m_2X\right]\left[1 + \beta\left(1 - \sqrt{1 - X^2}\right)\right]\bar{\phi}^2\,dXdY \qquad (20)$$

Here expression of the required frequency parameter is

$$\lambda^2 = \frac{12\rho_0 p^2 a^2(1 - \nu^2)}{E_0\bar{h}_0^2} \qquad (21)$$

Equation (18) contains two unknown constants, A_1 and A_2 which result from the substitution of deflection function $\phi(x, y)$.

The following formula could be used to determine these two unknowns:

$$\frac{\partial}{\partial A_n}\left[S_E^* - \lambda^2 K_E^*\right] = 0 \qquad (22)$$

After simplifying equation (22) we get system of homogeneous eq. as

$$C_{11}A_1 + C_{12}A_2 = 0$$

$$\text{and } C_{21}A_1 + C_{22}A_2 = 0 \qquad (23)$$

The determinant of the coefficient matrix obtained from equation (23) must be zero in order to produce a non-zero solution (frequency equation).

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0 \qquad (24)$$

After simplifying above equation we get a quadratic equation in λ. With λ representing frequency modes derived from equation (24), the time period of frequency modes is computed as $k = \frac{2\pi}{\lambda}$.

RESULT AND DISCUSSION

Duralumin, an aluminium alloy, is a visco-elastic material that produces the intended results. The calculations for Duralumin make use of the following parameters:

$E_0 = 7.08 \times 10^{10}$ N/M^2,

$G = 2.632 \times 10^{10}$ N/M^2,

$\eta = 14.612 \times 10^5$ N s/M^2,

$\rho_0 = 2.8 \times 10^3$ kg/M^3,

$\nu = 0.345$ and $h_0 = 0.01$ M

I) For aspect ratios of 1.5 and 2.5, calculations were carried out for the first two frequency modes for various values of the thermal gradient (α), non-homogeneity(m_2) and taper parameter (β).

II) The first two modes of the frequency parameter in Tables (1) and (4) increase continuously for both aspect ratios of 1.5 and 2.5 for every fixed value of the thermal gradient (α) while the taper parameter (β) increases from 0.2 to 0.6 and non-homogeneity (m_2) stays constant at $\nu = 0.345$. In every cases, the first two modes of the frequency parameter decrease as the thermal gradient (α) values rise from 0.0 to 0.8.

III) For any fixed value of non-homogeneity (m_2) in Tables (2) and (5), the first two modes of the frequency parameter increase steadily for both aspect ratios of 1.5 and 2.5 as the values of the thermal gradient (α) and taper parameter (β) rise from 0.2 to 0.8 with $\nu = 0.345$ In every scenario, the first two modes of the frequency parameter drop as the non-homogeneity (m_2) values rise from 0.0 to 1.0.

IV) In Table (3) and (6), for each fixed value of taper parameter(β), the first two mode of frequency parameter decrease continuously for both aspect ratio 1.5 and 2.5 as value of the thermal gradient (α) and non-homogeneity(m_2) increase from 0.2 to 0.6 with $\nu = 0.345$. As the values of the taper parameter(β) increases from 0.0 to 1.0, the first two mode of frequency parameter increases for all cases.

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Table1. Frequency of simple supported rectangular plate vs Thermal gradient(α) for Aspect Ratio 1.5

α	β = 0.2, m_2 = 0 ν=0.345		β = 0.4, m_2 = 0, ν=0.345		β = 0.6, m_2 = 0, ν=0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	57.6608	430.5953	60.2813	448.4819	63.0982	467.9346
0.2	56.2761	420.2152	58.9091	438.2127	61.7363	457.7718
0.4	54.8564	409.5722	57.5041	427.6970	60.3435	447.3783
0.6	53.3989	398.6451	56.0637	416.9161	58.9174	436.7375
0.8	51.9005	387.4099	54.5852	405.8490	57.4556	425.8310

Graphical representation of the table-1:

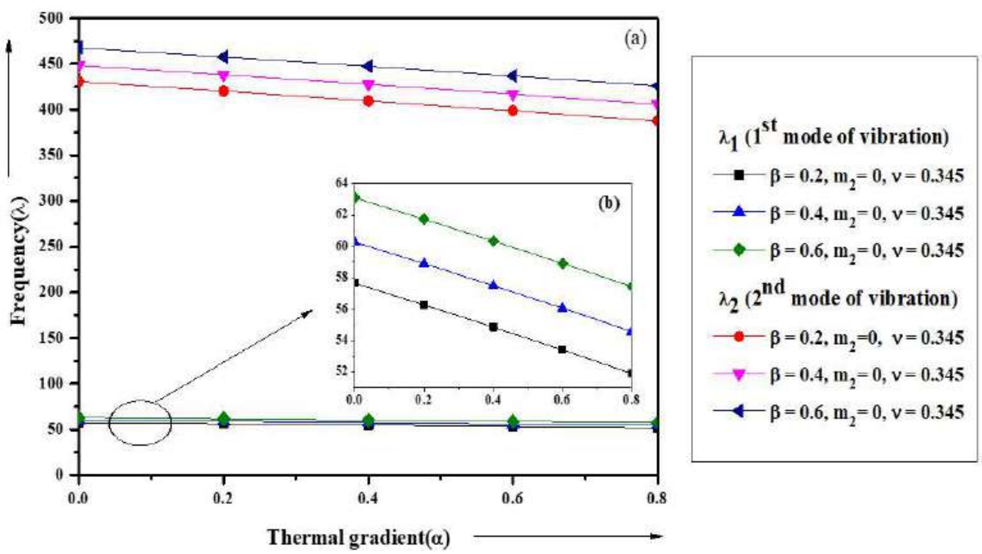

Figure-1: Thermal gradient vs Frequency

Table2. Frequency of simple supported rectangular plate vs Non-Homogeneity(m_2) for Aspect Ratio 1.5

m_2	α = β = 0.2, ν = 0.345		α = β = 0.4, ν = 0.345		α = β = 0.8, ν = 0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	56.2761	420.2152	57.5041	427.6970	60.4921	447.2278
0.2	53.6376	400.3576	54.7895	407.2026	57.6012	425.2683
0.4	51.3386	383.0730	52.4262	389.3980	55.0887	406.2570
0.6	49.3119	367.8497	50.3446	373.7419	52.8787	389.5878
0.8	47.5077	354.3082	48.4927	359.8344	50.9150	374.8161
1	45.8880	342.1599	46.8312	347.3723	49.1549	361.6071

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Graphical representation of the table-2:

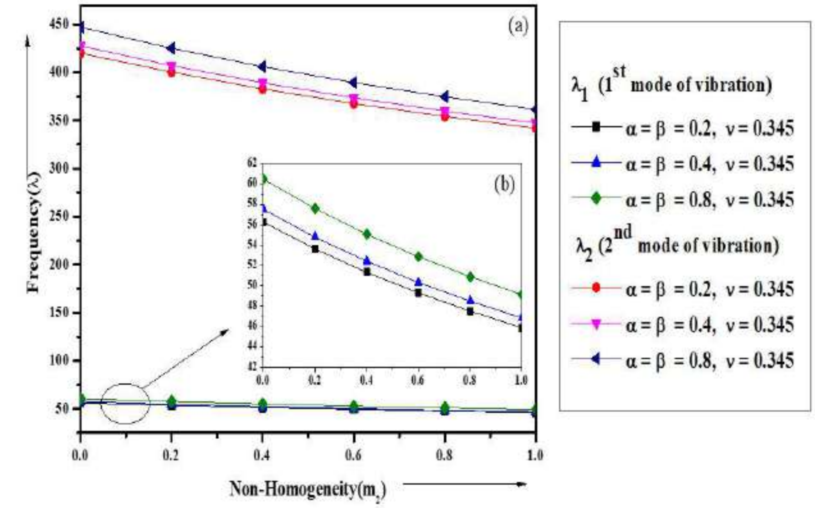

Figure-2: Non-Homogeneity vs Frequency

Table 3. Frequency of simple supported rectangular plate vs Taper constant(β) for Aspect Ratio 1.5

β	α = m_2 = 0.2, ν=0.345		α = m_2 = 0.4, ν=0.345		α = m_2 = 0.8, ν=0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	51.3499	385.0937	47.8525	358.8654	41.7691	313.2434
0.2	53.6376	400.3576	50.0434	373.3706	43.8139	326.6478
0.4	56.1282	417.2145	52.4262	389.3980	46.0313	341.4524
0.6	58.8033	435.5561	54.9830	406.8385	48.4039	357.5465
0.8	61.6449	455.2664	57.6962	425.5768	50.9150	374.8161
1	64.6361	476.2284	60.5493	445.4976	53.5492	393.1491

Graphical representation of the table-3:

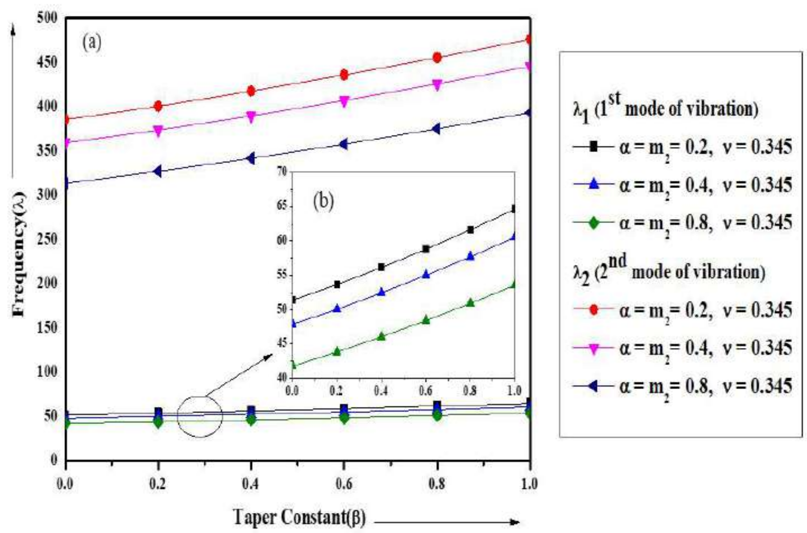

Figure-3: Taper Constant vs Frequency

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Table 4. Frequency of simple supported rectangular plate vs Thermal gradient(α) for Aspect Ratio 2.5

α	β = 0.2, m_2 = 0, ν=0.345		β = 0.4, m_2 = 0, ν=0.345		β = 0.6, m_2 = 0, ν=0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	46.1175	385.4861	47.9804	397.5019	49.9649	410.1599
0.2	44.9965	375.9693	46.8614	387.9363	48.8466	400.5400
0.4	43.8468	366.2052	45.7149	378.1289	47.7021	390.6833
0.6	42.6662	356.1736	44.5389	368.0602	46.5293	380.5715
0.8	41.4519	345.8511	43.3309	357.7083	45.3260	370.1835

Graphical representation of the table-4:

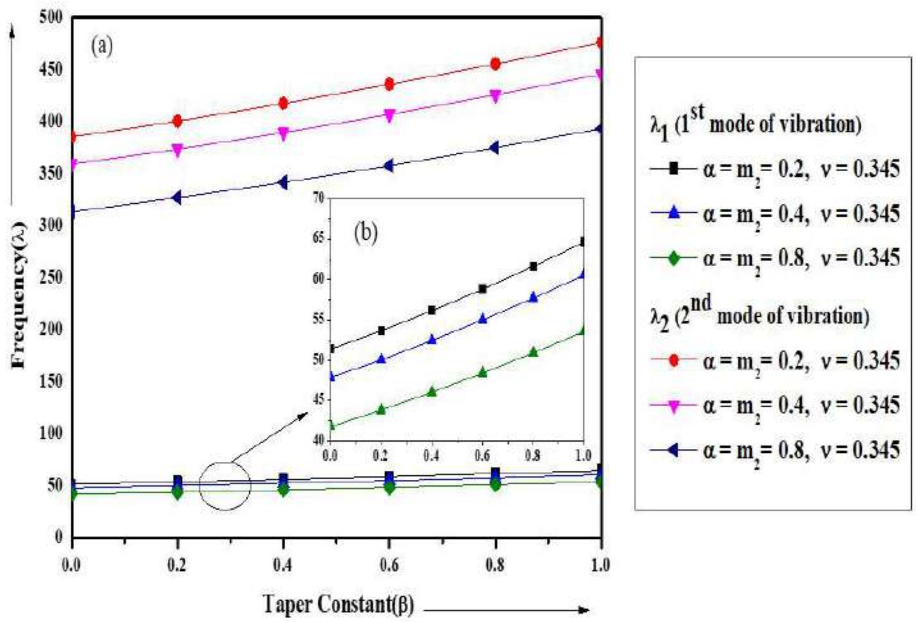

Figure-4: Thermal gradient vs Frequency

Table 5. Frequency of simple supported rectangular plate vs Non-Homogeneity(m_2) for Aspect Ratio 2.5

m_2	α = β = 0.2, ν = 0.345		α = β = 0.4, ν = 0.345		α = β = 0.8, ν = 0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	44.9965	375.9693	45.7149	378.1289	47.4270	383.2326
0.2	42.8868	358.2027	43.5567	360.0103	45.1601	364.4183
0.4	41.0486	342.7381	41.6779	344.2697	43.1900	348.1294
0.6	39.4281	329.1179	40.0230	330.4285	41.4571	333.8471
0.8	37.9855	317.0022	38.5508	318.1330	39.9173	321.1904
1	36.6905	306.1331	37.2299	307.1155	38.5373	309.8725

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Graphical representation of the table5:

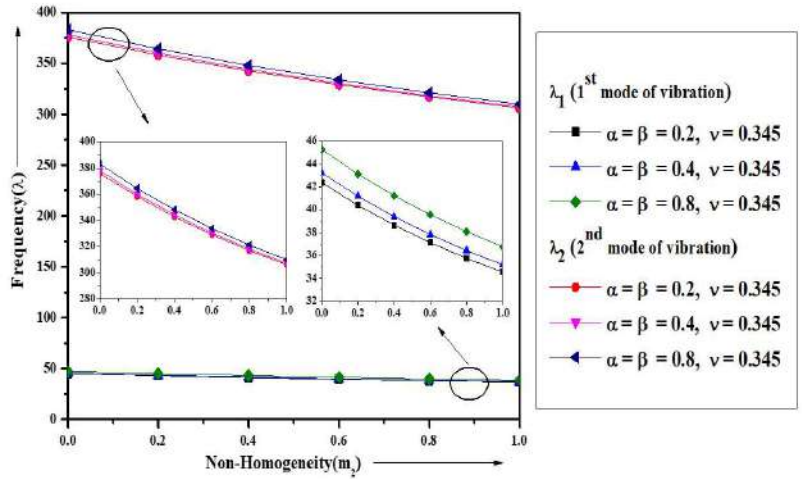

Figure-5: Non-Homogeneity vs Frequency

Table 6. Frequency of simple supported rectangular plate vs Taper constant(β) for Aspect Ratio 2.5

β	α = m_2 = 0.2, ν=0.345		α = m_2 = 0.4, ν=0.345		α = m_2 = 0.8, ν=0.345	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	41.2485	347.7121	38.4391	324.0298	33.5524	282.8364
0.2	42.8868	358.2027	39.9998	333.8371	34.9933	291.6077
0.4	44.6491	369.3479	41.6779	344.2697	36.5404	300.9521
0.6	46.5258	381.1032	43.4642	355.2836	38.1847	310.8269
0.8	48.5073	393.4249	45.3494	366.8358	39.9173	321.1904
1	50.5845	406.2707	47.3243	378.8849	41.7296	332.0032

Graphical representation of the table-6:

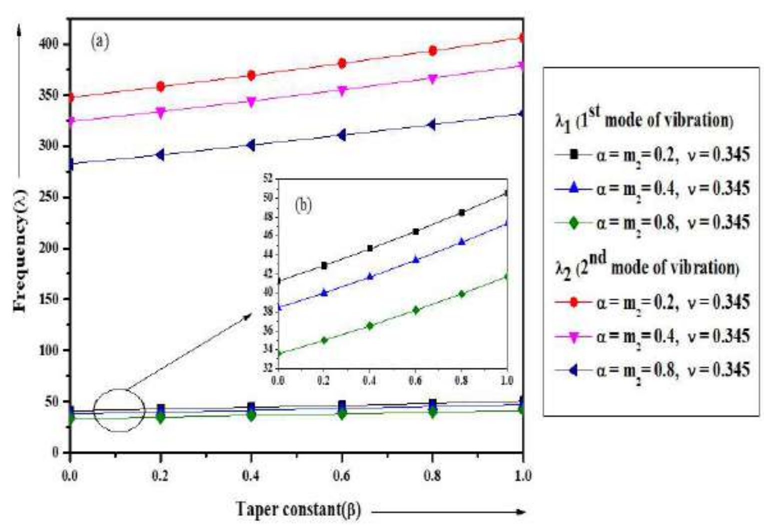

Figure-6: Taper constant vs Frequency

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CONCLUSION

The frequencies of isotropic rectangular plates with linear temperature and circular thickness and density fluctuations were examined in the current work. From the above result the increase in tapering constant (β) , results the increase in frequency λ_1 and λ_2 at different value of thermal gradient(α) and non-homogeneity (m_2). But increase in non-homogeneity (m_2) and thermal gradient (α), results the decrease in frequency. The variation in frequency mode λ_1 and λ_2 weather increasing or decreasing are very slow because the circular variation implementation. In the frequencies there is no quick increment or decrement.

REFERENCES

[1] W. Leissa, Vibration of plate, NASA SP-160(1969).

[2] Kumar, A., Lather, N., Bhardwaj, R., Mani, N. and Sharma, A., 2018. Effect of linear variation in density and circular variation in Poisson's ratio on time period of vibration of rectangular plate. Vibroengineering Procedia, 21, pp.14-19.

[3] Amabili, M., 2016. Nonlinear vibrations of viscoelastic rectangular plates. *Journal of Sound and Vibration*, *362*, pp.142-156.

[4] Khanna, A. and Sharma, A.K., 2013. Natural vibration of visco-elastic plate of varying thickness with thermal effect. Journal of Applied Science and Engineering, 16(2), pp.135-140.

[5] Khanna, A. and Singhal, A., 2016. Effect of plate's parameters on vibration of isotropic tapered rectangular plate with different boundary conditions. Journal of Low Frequency Noise, Vibration and Active Control, 35(2), pp.139-151.

[6] Kaur, N., Singhal, A. and Khanna, A., 2018. A study on vibration of tapered non-homogeneous rectangular plate with structural parameters. International Journal of Applied Mechanics and Engineering, 23(4), pp.873-884.

[7] Khanna, A. and Kaur, N., 2016. Effect of thermal gradient on vibration of non-uniform visco-elastic rectangular plate. Journal of The Institution of Engineers (India): Series C, 97, pp.141-148.

[8] Khanna, A. and Sharma, A.K., 2012. Mechanical vibration of visco-elastic plate with thickness variation. International Journal of Applied Mathematical Research, 1(2), pp.150-158.

[9] Manna, M.C., 2012. Free vibration of tapered isotropic rectangular plates. Journal of Vibration and Control, 18(1), pp.76-91.

[10] Dickinson, S.M., 1978. The buckling and frequency of flexural vibration of rectangular isotropic and orthotropic plates using Rayleigh's method. Journal of Sound and Vibration, 61(1), pp.1-8.

[11] Warburton, G.B., 1954. The vibration of rectangular plates. Proceedings of the Institution of Mechanical Engineers, 168(1), pp.371-384.

[12] Hussain, M. and Naeem, M.N., 2018. Effect of various edge conditions on free-vibration characteristics of isotropic square and rectangular plates. Advanced engineering testing.

[13] Bakhshandeh, A., Navayi Neya, B. and Nateghi Babagi, P., 2017. Benchmark solution for free vibration analysis of transversely isotropic thick rectangular plates. Acta Mechanica, 228, pp.3977-3995.

[14] Messina, A., 2011. Influence of the edge-boundary conditions on three-dimensional free vibrations of isotropic and cross-ply multilayered rectangular plates. Composite structures, 93(8), pp.2135-2151.

[15] Vanam, B.C.L., Rajyalakshmi, M. and Inala, R., 2012. Static analysis of an isotropic rectangular plate using finite element analysis (FEA). Journal of Mechanical Engineering Research, 4(4), pp.148-162.

[16] Li, W.L., 2004. Vibration analysis of rectangular plates with general elastic boundary supports. Journal of Sound and Vibration, 273(3), pp.619-635.

[17] Sharma, A., 2018. Vibrational frequencies of parallelogram plate with circular variations in thickness. In Soft Computing: Theories and Applications: Proceedings of SoCTA 2016, Volume 1 (pp. 317-326). Springer Singapore.

[18] Gupta, A.K. and Khanna, A., 2007. Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. Journal of Sound and Vibration, 301(3-5), pp.450-457.