



Scheduled various Maps regarding Beta-Sets and Allied Groups

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Abstract- The concept of group of functions, say $\beta\text{ch}(X, \tau)$ preserving β -closed sets containing homeomorphism group $h(X, \tau)$ was studied by Arora, Tahiliani and Maki. In continuation to that, we study some new isomorphisms, mappings, subgroups and their properties.

Keywords: α -open, β -open and β -irresolute mappings.

1. INTRODUCTION AND PRELIMINARIES

Throughout this paper we consider spaces on which no separation axiom are assumed unless explicitly stated. The topology of a space (By space we always mean a topological space) is denoted by τ and (X, τ) will be replaced by X if there is no chance of confusion. For $A \subseteq X$, the closure and interior of A in X are denoted by $\text{Cl}(A)$ and $\text{Int}(A)$ respectively. Let A be a subset of the space (X, τ) . Then A is said to be β -open [1] if $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$. Its complement is β -closed. The family of all β -open sets containing A is denoted by $\beta O(A)$ and all β -closed sets containing A is denoted by $\beta C(A)$. A is said to be α -open [6] if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$ and its complement is α -closed. The union of all β -open sets contained in A is called β -interior of A , denoted by $\beta \text{Int}(A)$ [2].

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called β -irresolute [4] if the inverse image of every β -open set in Y is β -open in X . It is called β c-homeomorphism [5] if f is β -irresolute bisection and f^{-1} is β -irresolute.

2. SUBGROUPS OF $\text{BCH}(X; \tau)$

For a topological space (X, τ) we have $h(X; \tau) = \{f \mid f : (X, \tau) \rightarrow (X, \tau) \text{ is a homeomorphism}\}$ [5] and $\beta\text{ch}(X; \tau) = \{f \mid f : (X, \tau) \rightarrow (X, \tau) \text{ is a } \beta\text{c-homeomorphism}\}$ [5].

In this section, we investigate some structures of $\beta\text{ch}(H; \tau|_H)$ for a subspace $(H, \tau|_H)$ of (X, τ) using two subgroups of $\beta\text{ch}(X, \tau)$, say $\beta\text{ch}(X, X \setminus H; \tau)$ and $\beta\text{ch}_0(X, X \setminus H; \tau)$ below.

Definition 2.1. For a topological space (X, τ) and subset H of X , we define the following families of maps:

- (i). $\beta\text{ch}(X, X \setminus H; \tau) = \{a \mid a \in \beta\text{ch}(X; \tau) \text{ and } a(X \setminus H) = X \setminus H\}.$
- (ii). $\beta\text{ch}_0(X, X \setminus H; \tau) = \{a \mid a \in \beta\text{ch}(X, X \setminus H; \tau) \text{ and } a(x) = x \text{ for every } x \in X \setminus H\}.$

Theorem 2.2. Let H be a subset of a topological space (X, τ) . Then

- (i) The family $\beta\text{ch}(X, X \setminus H; \tau)$ forms a subgroup of $\beta\text{ch}(X, \tau)$.
- (ii) The family $\beta\text{ch}_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta\text{ch}(X, X \setminus H; \tau)$ and hence $\beta\text{ch}_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta\text{ch}(X, \tau)$.

Proof. (i). It is shown obviously that $\beta\text{ch}(X, X \setminus H; \tau)$ is a non-empty subset of $\beta\text{ch}(X, \tau)$, because $1_X \in \beta\text{ch}(X, X \setminus H; \tau)$. Moreover, we have that $\omega_X(a, b^{-1}) = b^{-1} \circ a \in \beta\text{ch}(X, X \setminus H; \tau)$ for any elements $a, b \in \beta\text{ch}(X, X \setminus H; \tau)$, where $\omega_X = \omega|(\beta\text{ch}(X, X \setminus H; \tau) \times \beta\text{ch}(X, X \setminus H; \tau))$ as ω is the binary operation of the group $\beta\text{ch}(X, \tau)$. Evidently, the identity map 1_X is the identity element of $\beta\text{ch}(X, X \setminus H; \tau)$.

(ii). It is shown that $\beta\text{ch}_0(X, X \setminus H; \tau)$ is a non-empty subset of $\beta\text{ch}(X, X \setminus H; \tau)$ because $1_X \in \beta\text{ch}_0(X, X \setminus H; \tau)$. We have that $\omega_{X,0}(a, b^{-1}) = b^{-1} \circ a \in \beta\text{ch}_0(X, X \setminus H; \tau)$ for any elements $a, b \in \beta\text{ch}_0(X, X \setminus H; \tau)$, where $\omega_{X,0} = \omega_X|(\beta\text{ch}_0(X, X \setminus H; \tau) \times \beta\text{ch}_0(X, X \setminus H; \tau))$ (ω_X is the binary



operation of the group $\beta\text{ch}(X, X \setminus H; \tau)$. Thus $\beta\text{ch}_0(X, X \setminus H; \tau)$ is a subgroup of $\beta\text{ch}(X, X \setminus H; \tau)$ and the identity map 1_X is the identity element of $\beta\text{ch}_0(X, X \setminus H; \tau)$. By using (i), $\beta\text{ch}_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta\text{ch}(X, \tau)$. Let H and K be the subsets of X and Y respectively. For a map $f: X \rightarrow Y$ satisfying a property $K=f(H)$, we define the following map $r_{H,K}(f): H \rightarrow K$ by $r_{H,K}(f)(x)=f(x)$ for every $x \in H$. Then, we have that $j_K \circ r_{H,K}(f) = f|_H: H \rightarrow Y$, where $j_K: K \rightarrow Y$ be an inclusion defined by $j_K(y)=y$ for every $y \in K$ and $f|_H: H \rightarrow Y$ is a restriction of f to H defined by $(f|_H)(x)=f(x)$ for every $x \in H$. Especially, we consider the following case that $X=Y$, $H=K \subseteq X$ and $a(H)=H$, $b(H)=H$ for any maps $a, b: X \rightarrow X$. Thus $r_{H,H}(b \circ a) = r_{H,H}(b) \circ r_{H,H}(a)$ holds. Moreover, if a map $a: X \rightarrow X$ is a bisection such that $a(H)=H$, then $r_{H,H}: H \rightarrow H$ is bijective and $r_{H,H}(a^{-1}) = (r_{H,H}(a))^{-1}$.

We recall well known properties on β -open sets of subspace topological spaces.

Theorem 2.3. For a topological space (X, τ) and subsets H and U of X and $A \subseteq H$, $V \subseteq H$ and $B \subseteq H$, the following properties hold:

(i). Arbitrary union of β -open sets of (X, τ) is β -open in (X, τ) . The intersection of an open set of (X, τ) and a β -open set in (X, τ) is β -open in (X, τ) .

(ii). (a). If A is β -open in (X, τ) and $A \subseteq H$, then A is β -open in a subspace $(H, \tau|_H)$.

(b). If $H \subseteq X$ is open or α -open in (X, τ) and a subset $U \subseteq X$ is β -open in (X, τ) , then $H \cap U$ is β -open in subspace $(H, \tau|_H)$.

(iii). Let $V \subseteq H \subseteq X$.

(a). If H is β -open in (X, τ) , then $\text{Int}_H(V) \subseteq \beta\text{Int}(V)$ holds.

(b). If H is β -open in (X, τ) and V is β -open in a subspace $(H, \tau \bullet H)$ then V is β -open in (X, τ) . Let $B \bullet H \bullet X$. If H is β -closed in (X, τ) and B is β -closed in a subspace $(H, \tau \bullet H)$, then B is β -closed in (X, τ) .

(a). Assume that H is a open subset of (X, τ) . Then, $\beta O(X, \tau) \bullet H \bullet \beta O(H, \tau \bullet H)$ holds, where $\beta O(X, \tau) \bullet H = \{W \bullet H \bullet W \bullet \beta O(X, \tau)\}$.

(b). Assume that H is a β -open subset of (X, τ) . Then, $\beta O(H, \tau \bullet H) \bullet \beta O(X, \tau) \bullet H$ holds.

(c). Assume that H is a β -open subset of (X, τ) . Then, $\beta O(H, \tau \bullet H) = \beta O(X, \tau) \bullet H$ holds.

Proof. (i). Clear from Remark 1.1 of [1] and Theorem 2.7 of [3].

(ii). (a). Clear. (ii-2). Its Lemma 2.5 of [1].

(iii). (a). Let $x \bullet \text{Int}_H(V)$. There exists a subset $W(x) \in \tau$ such that $W(x) \cap H \subseteq V$. By (i), $W(x) \cap H \in \beta O(X, \tau)$. This shows that $x \in \beta\text{Int}(V)$ and so $\text{Int}_H(V) \subseteq \beta\text{Int}(V)$.

(b) and (iv). Its clear from Lemma 2.7 of [1].

(v). (b). Let $V \in \beta O(X, \tau)|_H$. For some set $W \in \beta O(X, \tau)$, $V = W \cap H$ and so we have $W \cap H \in \beta O(H, \tau|_H)$ (from ii-2). Hence $V \in \beta O(H, \tau|_H)$ holds.

(b). Let $V \in \beta O(H, \tau|_H)$. Since $H \in \beta O(X, \tau)$, we have $V \in \beta O(X, \tau)$ by (iii-2). Thus $V = V \cap H \in \beta O(X, \tau)|_H$.

(c). It follows from (v-1) and (v-2).

Lemma 2.4. (i). If $f: (X, \tau) \rightarrow (Y, \sigma)$ is β -irresolute and a subset H is α -open in (X, τ) , then $f|_H: (H, \tau|_H) \rightarrow (Y, \sigma)$ is β -irresolute. Let (1) and (2) be properties of two maps $k: (X, \tau) \rightarrow (K, \sigma|_K)$, where $K \subseteq Y$, and $j_K \circ k: (X, \tau) \rightarrow (Y, \sigma)$ as follows:

1) $k: (X, \tau) \rightarrow (K, \sigma|_K)$ is β -irresolute.

2) $j_K \circ k: (X, \tau) \rightarrow (Y, \sigma)$ is β -irresolute.

Then, the following implication and equivalence hold:

(a). Under the assumption that K is α -open in (Y, σ) , (1) $\Rightarrow$ (2).

(b). Conversely, under the assumption that K is β -open in (Y, σ) , (2) $\Rightarrow$ (1). (ii-3). Under the assumption that K is β -open in (Y, σ) , (1) $\Leftrightarrow$ (2).

(ii). If $f: (X, \tau) \rightarrow (Y, \sigma)$ is β -irresolute and a subset H is α -open in (X, τ) and $f|_H$ is β -open in (Y, σ) , then $r_{H,f(H)}(f)$:



$(H, \tau|_H) \rightarrow (f(H), \sigma|_{f(H)})$ is β -irresolute.

Proof. (i). Let $V \in \beta O(Y, \sigma)$. Then, we have $(f|_H)^{-1}(V) = f^{-1}(V) \cap H$ and $(f|_H)^{-1}(V) \in \beta O(H, \tau|_H)$. (Theorem 2.3 (ii-2)).

(ii). (a) $(1) \Rightarrow (2)$. Let $V \in \beta O(Y, \sigma)$. Since $(jKok)^{-1}(V) = k^{-1}(V \cap K)$ and $V \cap K \in \beta O(K, \sigma|_K)$ (Theorem 2.3 (ii-2)), we have that $(jKok)^{-1}(V) \in \beta O(X, \tau)$ and hence $jKok$ is β -irresolute.

(b) $(2) \Rightarrow (1)$. Let $U \in \beta O(K, \sigma|_K)$. Since $U \in \beta O(Y, \sigma)$ (Theorem 2.3 (iii-2)), we have $k^{-1}(U) = (jKok)^{-1}(U) \in \beta O(X, \tau)$. Thus k is β -irresolute.

(c). Obvious in the view of fact that every α -open set is β -open, it is obtained by (ii-1) and (ii-2).

(iii). By (i), $f|_H: (H, \tau|_H) \rightarrow (Y, \sigma)$ is β -irresolute. The map $r|_H: f(H), \sigma|_{f(H)}$ is β -irresolute, because $f|_H = jf(H) \circ r|_H, f(H), \sigma|_{f(H)}$ holds.

Definition 2.5. For an α -open subset H of (X, τ) , the following maps $(rH)^*: \beta ch(X, X \setminus H; \tau) \rightarrow \beta ch(H; \tau|_H)$ and $(rH)^*_0: \beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch_0(H; \tau|_H)$ are well defined as follows (Lemma 2.4 (iii)), respectively:

$(rH)^*(f) = r|_H, H(f)$ for every $f \in \beta ch(X, X \setminus H; \tau)$;

$(rH)^*_0(g) = r|_H, H(g)$ for every $g \in \beta ch_0(X, X \setminus H; \tau)$. Indeed, in Lemma 2.4 (iii), we assume that $X=Y, \tau=\sigma$ and $H=f(H)$.

Then, under the assumption that H is α -open hence β -open in (X, τ) , it is obtained that $r|_H, H(f) \in \beta ch(H; \tau|_H)$ holds for any $f \in \beta ch(X, X \setminus H; \tau)$ (resp. $f \in \beta ch_0(X, X \setminus H; \tau)$).

We need the following lemma and then we prove that $(rH)^*$ and $(rH)^*_0$ are onto homomorphisms under the assumptions that H is α -open and α -closed in (X, τ) .

Let $X=U_1 \cup U_2$ for some subsets U_1 and U_2 and $f_1: (U_1, \tau|_{U_1}) \rightarrow (Y, \sigma)$ and

$f_2: (U_2, \tau|_{U_2}) \rightarrow (Y, \sigma)$ be the two maps satisfying a property $f_1(x) = f_2(x)$ for every $x \in U_1 \cap U_2$. Then, a map $f_1 \vee f_2$ is well defined as follows:

$(f_1 \vee f_2)(x) = f_1(x)$ for every $x \in U_1$ and $(f_1 \vee f_2)(x) = f_2(x)$ for every $x \in U_2$. We call this map a combination of f_1 and f_2 .

Lemma 2.6. For a topological space (X, τ) , we assume that $X = U_1 \cup U_2$, where U_1 and U_2 are subsets of X and $f_1: (U_1, \tau|_{U_1}) \rightarrow (Y, \sigma)$ and $f_2: (U_2, \tau|_{U_2}) \rightarrow (Y, \sigma)$ be the two maps satisfying a property $f_1(x) = f_2(x)$ for every $x \in U_1 \cap U_2$. Then if $U_i \in \beta O(X, \tau)$ for each $i \in \{1, 2\}$ and f_1 and f_2 are β -irresolute, then its combination $f_1 \vee f_2: (X, \tau) \rightarrow (Y, \sigma)$ is β -irresolute.

Proof. Its on similar lines in ([1], Theorem 2.8).

Theorem 2.7. Let H be a subset of a topological space (X, τ) .

(i). (a). If H is α -open in (X, τ) , then the maps $(rH)^*: \beta ch(X, X \setminus H; \tau) \rightarrow \beta ch(H; \tau|_H)$ and $(rH)^*_0: \beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch_0(H; \tau|_H)$ are

homomorphism of groups. Moreover $(rH)^*|_{\beta ch_0(X, X \setminus H; \tau)} = (rH)^*_0$ holds (Definition 2.5).

(b). If H is α -open and α -closed in (X, τ) , then the maps $(rH)^*: \beta ch(X, X \setminus H; \tau) \rightarrow \beta ch(H; \tau|_H)$ and $(rH)^*_0: \beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch_0(H; \tau|_H)$ are onto homomorphism of groups.

(ii). For an α -open subset H of (X, τ) , we have the following isomorphisms of groups: (ii-1). $\beta ch(X, X \setminus H; \tau) / \text{Ker}(rH)^*$ is isomorphic to $\text{Im}(rH)^*$;

(b). $\beta ch_0(X, X \setminus H; \tau)$ is isomorphic to $\text{Im}(rH)^*_0$, 0 holds. where $\text{Ker}(rH)^* = \{a \in \beta ch(X, X \setminus H; \tau) \mid (rH)^*(a) = 1_X\}$ is a normal subgroup of $\beta ch(X, X \setminus H; \tau)$; $\text{Im}(rH)^* = \{(rH)^*(a) \mid a \in \beta ch(X, X \setminus H; \tau)\}$ and $\text{Im}(rH)^*_0 = \{(rH)^*_0(b) \mid b \in \beta ch_0(X, X \setminus H; \tau)\}$ are subgroups of $\beta ch(X, \tau)$.

(iii). For an α -open and α -closed subset H of (X, τ) , we have the following isomorphisms of groups:

(c). $\beta ch(H; \tau|_H)$ is isomorphic to $\beta ch(X, X \setminus H; \tau) / \text{Ker}(rH)^*$. (iii-2). $\beta ch_0(H; \tau|_H)$ is isomorphic to $\beta ch_0(X, X \setminus H; \tau)$.

Proof. (i).

(a). Let $a, b \in \beta ch(X, X \setminus H; \tau)$. Since H is α -open in (X, τ) , the maps $(rH)^*$ and $(rH)^*_0$ are well defined (Definition 2.5). Then we have that $(rH)^*(\omega_X(a, b)) = (rH)^*(boa) = r|_H, H(boa) = r|_H, H(b) \circ r|_H, H(a) = \omega_X((rH)^*(a), (rH)^*(b))$ hold, where ω_H is a binary operation of $\beta ch(H; \tau|_H)$ ([5] Theorem 4.4 (iv)). Thus $(rH)^*$ is a homomorphism of groups. For the map $(rH)^*_0: \beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch_0(H; \tau|_H)$, we



have that $(rH)^*, 0 (\omega X, 0(a, b)) = (rH)^*, 0 (boa) = rH, H(boa) = rH, H(b) \circ rH, H(a) = \omega X((rH)^*(a), (rH)^*(b))$ hold, where ωX is a binary operation of $\beta ch(H; \tau|H)$ (Theorem 2.3 (ii)). Thus $(rH)^*, 0$ is also a homomorphism of groups. It is obviously shown that $(rH)^* | \beta ch_0(X, X \setminus H; \tau) = (rH)^*, 0$ holds. (Definitions 2.1 and 2.5).

(b). In order to prove that $(rH)^*$ and $(rH)^*, 0$ are onto, let $h \in \beta ch(H; \tau|H)$. Let $jH: (H; \tau|H) \rightarrow (X, \tau)$ and $JX \setminus H: (X \setminus H, \tau|X \setminus H) \rightarrow (X, \tau)$ be the inclusions defined $jH(x) = x$ for every $x \in H$ and $JX \setminus H(x) = x$ for every $x \in X \setminus H$. We consider the combination $h_1 = (jH \circ h) \nabla (jX \setminus H \circ 1_{X \setminus H}): (X, \tau) \rightarrow (X, \tau)$. By Lemma 2.4 (ii-1), under the assumption of α -openness on H , it is shown that two maps $jH \circ h: (H; \tau|H) \rightarrow (X, \tau)$ and $jH \circ h^{-1}: (H; \tau|H) \rightarrow (X, \tau)$ are β -irresolute; moreover under the assumption of α -openness on $X \setminus H$, $JX \setminus H \circ 1_{X \setminus H}: (X \setminus H, \tau|X \setminus H) \rightarrow (X, \tau)$ is β -irresolute. Using lemma 2.6, for a β -open cover $\{H, X \setminus H\}$ of X , the combination above $h_1: (X, \tau) \rightarrow (X, \tau)$ is β -irresolute. Since h_1 is bijective, its inverse map $h_1^{-1} = (jH \circ h^{-1}) \nabla (jX \setminus H \circ 1_{X \setminus H})$ is also β -irresolute. Thus under the assumption that both H and $X \setminus H$ are β -open in (X, τ) , we have $h_1 \in \beta ch(X, \tau)$. Since $h_1(x) = x$ for every point $x \in X \setminus H$, we conclude that $h_1 \in \beta ch_0(X, X \setminus H; \tau)$ and so $h_1 \in \beta ch(X, X \setminus H; \tau)$. Moreover, $(rH)^*, 0 (h_1) = (rH)^*(h_1) = rH, H(h_1) = h$, hence $(rH)^*$ and $(rH)^*, 0$ are onto, under the assumption that H is α -open and α -closed subset of (X, τ) .

(ii). By (a) above and the first isomorphism theorem of group theory, it is shown that there are group isomorphism below, under the assumption that H is α -open in (X, τ) : (*). $\beta ch(X, X \setminus H; \tau) | \text{Ker}(rH)^*$ is isomorphic to $\text{Im}(rH)^*$; and (**). $\beta ch_0(X, X \setminus H; \tau) | \text{Ker}(rH)^*, 0$ is isomorphic to $\text{Im}(rH)^*, 0$ where $\text{Ker}(rH)^*, 0 = \{a \in \beta ch_0(X, X \setminus H; \tau) | (rH)^*, 0(a) = 1_X\}$. Moreover, under the assumption of α -openness on H , it is shown that $\text{Ker}(rH)^*, 0 = \{1_H\}$. Therefore, using (**) above, we have the isomorphism (ii-2).

(iii). By (b) above, it is shown that $(rH)^*$ and

$(rH)^*, 0$ are onto homomorphism of groups, under the assumption that H is α -open and α -closed in (X, τ) . Therefore, by (ii) above, the isomorphisms (iii-1) and (c) are obtained.

Remark 2.8. Under the assumption that H is α -open and α -closed in (X, τ) , Theorem 2.7 (iii) is proved. Let (X, τ) be a topological space where $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, and $(H; \tau|H)$ is a subspace of (X, τ) , where $H = \{a\}$. Then $\beta O(X, \tau) = P(X)$ (the power set of X) and H is α -open and α -closed in (X, τ) . We apply Theorem 2.7 (iii) to the present case, we have the group isomorphisms. Directly, we obtain the following data on groups: $\beta ch(X, \tau)$ is isomorphic to S_3 , the symmetric group of degree 3, $\beta ch(X, X \setminus H; \tau) = \{1_X, h_a\}$, $\text{Ker}(rH)^* = \{1_X, h_a\}$, $\beta ch(H; \tau|H) = \{1_H\}$ and so $\beta ch_0(X, X \setminus H; \tau) = \{1_X\}$, where $h_a: (X, \tau) \rightarrow (X, \tau)$ is a map defined by $h_a(a) = a$, $h_a(b) = c$ and $h_a(c) = b$. Therefore in this example, we have $\beta ch(H; \tau|H)$ is isomorphic to $\beta ch(X, X \setminus H; \tau)$ $|\text{Ker}(rH)^*$ and $\beta ch(H; \tau|H)$ is isomorphic to $\beta ch_0(X, X \setminus H; \tau)$. Moreover we have $h(X, \tau) = \{1_X, h_a\}$.

(iii). Even if a subset H of a topological space (X, τ) is not α -closed and it is α -open, we have the possibilities to investigate isomorphisms of groups corresponding to a subspace $(H, \tau|H)$ and $(rH)^*$ using Theorem 5.7 (ii). For example, Let (X, τ) be a topological space where $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a, b\}\}$, and $(H; \tau|H)$ is a subspace of (X, τ) , where $H = \{a, b\}$. Then $\beta O(X, \tau) = P(X)$ (the power set of X) and H is α -open but not α -closed in (X, τ) . By theorem 2.7 (i)-(1), the maps $(rH)^*: \beta ch(X, X \setminus H; \tau) \rightarrow \beta ch(H; \tau|H)$ and $(rH)^*, 0: \beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch(H; \tau|H)$ are homomorphism of groups and by theorem 5.7 (ii) two isomorphisms of groups are obtained:

(*-1). $\beta ch(X, X \setminus H; \tau) / \text{Ker}(rH)^*$ is isomorphic to $\text{Im}(rH)^*$. (*-2). $\beta ch_0(X, X \setminus H; \tau) / \text{Ker}(rH)^*, 0$ is isomorphic to $\text{Im}(rH)^*, 0$.

We need notation on maps as follows: let $h_c: (X, \tau) \rightarrow (X, \tau)$ and $t_{a,b}: (H, \tau|H) \rightarrow (H, \tau|H)$ are



the maps defined by $h_c(a)=b$, $h_c(a)=b$, $h_c(c)=c$ and $t_{a,b}(a)=b$, $t_{a,b}(b)=a$, respectively. Then it is directly shown that $\beta_{ch}(X, X \setminus H; \tau) = \{1_X, h_c\}$ which is isomorphic to Z_2 , $(h_c)^2 = 1_X$, and $\text{Ker}(\tau H)^* = \{a \in \beta_{ch}(X, X \setminus H; \tau) \mid (\tau H)^*(a) = 1_H\} = \{a \in \{1_X, h_c\} \mid (\tau H)^*(a) = 1_H\} = \{1_X\}$ because $(\tau H)^*(1_X) = 1_H$ and $(\tau H)^*(h_c) = t_{a,b} \neq 1_H$. By using $(*)$ above, $\text{Im}(\tau H)^*$ is isomorphic to $\beta_{ch}(X, X \setminus H; \tau) = \{1_X, h_c\}$ and so $\text{Im}(\tau H)^* = \{1_H, \tau H, H(h_c)\} = \{1_H, t_{a,b}\}$. Since $\text{Im}(\tau H)^* \subseteq \beta_{ch}(H; \tau|_H) \subseteq \{1_H, t_{a,b}\}$, we have that $\text{Im}(\tau H)^* = \beta_{ch}(H, \tau|_H) = \{1_H, t_{a,b}\}$ and hence $(\tau H)^*$ is onto. Namely, we have an isomorphism $(\tau H)^*: \beta_{ch}(X, X \setminus H; \tau)$ is isomorphic to $\beta_{ch}(H; \tau|_H)$ which is isomorphic to Z_2 . Moreover it is shown that $\beta_{ch_0}(X, X \setminus H; \tau) = \{a \in \beta_{ch}(X, X \setminus H; \tau) \mid a(x) = x \text{ for any } x \in \{c\}\} = \{1_X, h_c\} = \beta_{ch}(X, X \setminus H; \tau)$ hold and so $(\tau H)^* = (\tau H)^*_0$ holds.

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