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Study and Analysis of Dynamical Models of Plant Growth Analysis in Calculus

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Abstract- This paper presents the study of dynamical models of plant growth analysis in calculus. It also the studies modeling of plants, dynamical models, first dimensional model with branching, without branching and their results.

Keywords: Plants, growths, model, dynamical, one-dimensional

1. INTRODUCTION

1.1 Modelling plant growth

In the middle ages, Leonardo da Vinci observed the seasonal periodicity of growth and some features of plant forms [1].Theories of phyllotaxis, which can be defined as "a construction determined by organs, parts of organs, or primordial of plants" [2] appear already in the 17th century. D' Arcy Thompson reviewed early theories [1] and R.V. Jean contemporary theories of phyllotaxis [2].The best mechanism of pattern formation in mathematical biology is related to reaction - diffusion systems and Turing structures [3], [4], and the discussion below. However, there is no biological evidence that this mechanism is really involve in biological pattern formation [2], [5].Some otherapproaches use the optimization mechanism. For example, the branching pattern in plants can be related to maximization of light interception [6]. Plant topology and design are studied in [7], [8]. Some other aspects of plant modeling can be found in the Proceedings of the Workshop on Plant Models [9]. There are several recent experimental works that establish a relationship between expression of certain genes and formation of plant organs [10]. If L (t) is the plant size that depends on time t, then we can consider the empirical equation.

$$\frac{dL}{dt} = F(L),$$

Where F can be proportional to L (autocatalytic growth), or be some constant (linear growth, of $F(L) = aL(L_0 - L)$, where a and L_o are parameters [11], [12]. Such kinetic equations have been proposed since the early 20th century [1], with no significant progress since then. It is interesting to note that D'Arcy Thompson discusses autocatalytic growth in relation to chemical kinetics and plant hormones.

1.2 Dynamic model

One of the most important features of plants, for purposes of modelling them, is that proliferating cells are strongly localized. The growing part of the plant where cells divide is called the meristem. The primary or apical meristem is located at the very end of growing shoots and represents a narrow layer of cells with a more or less constant width for each particular plant. The biological mechanism that provides the localization of the apical meristem is related to the expression of certain genes [13].

Some parts of the primary meristem can remain in the internodes. Under certain conditions, determined by plant hormones, they can lead to the appearance of buds that can develop into branches. The secondary meristem, or cambium, is responsible for width-wise growth of the plant.

If we consider only the apical meristem, then we can say that cell proliferation and growth determine plant growth. Outside this narrow layer, cells differentiate; they cannot divide any more, and they serve to conduct biological products. Cell division and growth are controlled by external signals called growth and mitosis factors. Each of them is a generic name for a number of biological products. In particular, mitosis factors tell the cell when it should go from a rest state, where it can remain an indefinitely long time, to a division cycle. In some cases, the same molecule can play both roles. In what follows we will not distinguish between these two factors and will call them for brevity GM-factors. They are produced in meristemic cells and can be transmitted between neighboring cells.

The relatively simple structure of plants, where the growing part is strongly localized, suggests very natural mathematical models describing their growth. We describe plant growth with free boundary problems where the motion of the interface corresponds to the displacement of the apical

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meristem. The speed of the growth,that is of the interface motion, is determined by diffusion and convective fluxes of nutrients in the plant and by a self-accelerating production of plant growth factors in the meristem [14].

Thus the model suggested in this work is based on the following biological facts and mathematical approximations:

a). The growing part of the plant, or apex, contains a narrow exterior part, the meristem where cells proliferate providing the plant growth. This layer has a constant width and consists of an approximately constant number of cell layers specific to each plant. Since it is very small compared to the whole plant, it will be considered as a mathematical surface. The displacement of this surface corresponds to the plant growth.

b). The appearance of new cells implies that old cells exit this external layer after some time and become a part of the internal plant tissue. They differentiate, that is, they change their functions. They cannot divide any more, and they serve to conduct nutrients to the meristem.

c). The proliferation rate is determined by the concentration of nutrients and of GM-factors in the meristem. The GM-factors are produced in the meristem. The rate of their production is self-accelerationg. They can be transmitted between neighboring cells [13].

d). Appearance of new buds is determined by concentrations of certain plant hormones [11], [15]. The hormones are produced either in the growing parts of the plant (in our case in the meristem, there are no leaves in the model), or in the root and supplied to the plant above with the flow of nutrients.

Some more specific details of the model will be discussed below.

1.3. Results: 1D case, In the next section we study a one-dimensional model without branching. The growing plant is represented as an interval with its left end point fixed at $x = 0$ and its right end point at $x = L(t)$. The length $L(t)$ is a function of time. Nutrients enter through $x = 0$ and are transported through the interval by convective and diffusive fluxes. The speed of growth $V(t) = L'(t)$ depends on the concentration C of nutrients and on the concentration $R(t)$ of the GM-factor at $x = L(t)$. The production of the GM-factor is described by the equation

$$\frac{dR}{dt} = Cg(R) - \sigma R \qquad (1.2)$$

The typical form of the function $g(R)$ is shown in Figure 1 though we often use a smooth function, σ is a parameter. Its first derivative increases at some interval of R. This allows us to describe an auto-catalytic production of the GM-factor. The second term in the right-hand side of this equation describes consumption or destruction of the factor.

Another essential property of the function $g(R)$ is related to the value of its derivative at $R = 0$. Assuming that the dimensionless concentration C changes between 0 and 1 with $C = 1$ at $x = 1$, we choose $g'(0)$ slightly greater than σ. Therefore, if the concentration C of nutrients at the growing end is small, then the GM-factor will not be produced. Moreover, its concentration will be decreasing. If C is close to its maximal value, then the right-hand side in (1.2) becomes positive, and the concentration of the GM-factor will grow.

The growth rate V is considered as a given function of the GM-factor, $V = f(R)$. For simplicity, we suppose that it zero for $R \le R_1 \ge R_0$. Thus, the rate of plant growth equals zero for small concentrations of the GM-factor, and some positive constant for large concentrations.

In the oscillating mode, periods of growth alternate with periods of rest. During periods of growth, the nutrients are consumed and the concentration of the GM-factor in the meristem is high. During the periods of rest, the concentration of the GM-factor is low, and the concentration of nutrients increases. The number of periods of growth strongly depends on the parameters. It can vary from one to probably infinity. After a number of periods of growth a steady state is reached, and the length L(t) does not change any more.

The increase in length is approximately the same during each period of growth. The final plant length is determined by the number of growth periods. The periods of rest increase with time since a larger plant needs more time to transfer nutrients from the root to the meristem. Oscillations in plant growth can be related to endogenous rhythms, i.e., the rhythms that occur under constant external conditions.

We briefly explain the mechanism of the oscillations. It can be verified that there exist two continuous families of stationary solutions; stable and unstable (see Annexe 3). The solution of the evolution problem approaches first an unstable solution along its stable manifold and then diverges from it along its unstable manifold. The it approaches in the same way another unstable solution and so on. After several such cycles it finally approaches a stable stationary solution and does not changes after that. The number of cycles depends on the parameters and on the initial conditions.

2. 1D CASE WITH BRANCHING

We model here a growing plant as a system of intervals, which we will can branches. The number

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and location of branches is not given a priori. They will appear and grow according to some rules. In fact, each branch grows according to the same mechanism described above in the one-dimensional case without branching. The difference is that all branches except for the first one start from another branch and not from the root. The opposite end of the branch corresponds to the apical meristem. The end point of each branch has its own value of GM-factor concentration described by the equation similar to (1.2).

We need to impose two additional conditions on the concentration of nutrients at the points of branching, that is, the point where another branch starts. The first one is the continuity of the concentration, and the second one is the conservation of fluxes (see Section 4). To write this relation, we need to know the relation between cross section areas of branches below and above the branching point. This question also represents an independent interest. It appears that there is conservation of cross section areas asymptotically for a long time. It is satisfied with good accuracy in actual trees.

Appearance of new branches in the model occurs according to the following mechanism. First, there is a new bud that can appear if some conditions on the concentrations of plant hormones are satisfied. The bud is considered as a small branch: it is connected to another branch by one end point, and it has is apical meristem at the other end point. The distribution of nutrient inside the bud and the value of the GM-factor determine whether it starts growing.

We consider two plant hormones in the model. One of them corresponds to auxin and another one to cytokinin. Both are produced in the growing parts of the plant, in our case in the apical meristem, and are transported through the whole plant. It is known that these two hormones play an important role in formation of new buds (see [11]). However, the specific form of the branching condition is not known. We discuss this question in Section 4 and suggest branching conditions which seem to give the results in agreement with biological observations.

We observe a wide variety of plant forms and study more specifically the question of apical domination.

3. 1D MODEL WITHOUT BRANCHING

3.1. Model

We consider in this section the one-dimensional case justified if the length (or height) L of the plant is essentially greater than the diameter of its trunk. Hence we consider the interval $0 \le x \le L(t)$ with the length depending on time. The left endpoint $x = 0$ corresponds to the root. Its role is to provide the flux of nutrients taken into account through the boundary condition. We do not model the root growth here. Therefore the left boundary is fixed. The right end point, $x = L(t)$ corresponds to the apex. Its width is much less than that of the plant. We suppose in the model that it is a mathematical point. The value $L(t)$ increases over time. According to the assumption above, the growth rate is determined by the concentration of metabolites at $x = L(t)$, which we denote by R. Thus

$$\frac{dL}{dt} = f(R) \qquad (2.1)$$

The function f(R) will be specified below.

We recall that the interval $0 < x < L(t)$ corresponds to differentiated cells that conduct nutrients from the root to the apex. We suppose that they are in a liquid solution. Denote by C their concentration, which is a function of x and t. Its evolution is described by the diffusion-advection equation.

$$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial c} = d\,\frac{\partial^2 C}{\partial x^2}. \qquad (2.2)$$

Here u is the velocity of the fluid, and d is the diffusion coefficient. Assuming that the fluid is incompressible and fills the xylem uniformly (the part of the plant tissue conducting nutrients from below to above and located inside the cambium layer), we obtain

We complete equation (2.2) by setting the boundary conditions

$$x = 0 : C = 1, x = L(t) : d\,\frac{\partial C}{\partial x} = -g(R)C. \qquad (2.3)$$

The second boundary condition shows that the flux of nutrients from the main body of the plant to the meristem is proportional to the concentration C (L, t). This is a conventional relation for mass exchange at the boundary, Robin boundary conditions. The factor g(R) shows that this flux can be regulated by proliferating cells. We discuss this assumption as well as the form of the function g(R) below.

We now derive the equation describing the evolution of R. At this point we need to return to the model in which the width of the meristem is finite. We denote it by h. Then we have

$$h\,\frac{dR}{dt} = g(R)C - {}_{\partial} R. \qquad (2.4)$$

The first term in the right-hand side of this equation describes production of the GM-factor R in the meristem. The second term corresponds to its consumption.

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System of equations (2.1)-(2.4) is a generic one-dimensional model of plant growth based on:

a) "continuous medium" assumptions of mass conservation (for $C + R$) and of the proportionality of the flux $\partial C / \partial x$ at the boundary to the value of C; and b) a "biological" assumption that there is a chemical species R, the plant growth and mitosis factor, which is produced in the meristem and which determines the plant growth.We now specify the form of the functions f and g. We will consider f as a piecewise constant function equal to 0 if R is less than a critical value R_f and equal to some positive constant f_0 if R is greater than R_f (figure 1a). This means that the growth begins if the concentration of the plant growth factor exceeds some critical value.

The production of the growth factor R is assumed to be auto-catalytic. To simplify the modelling we consider a piece-wise linear function g(R) (Figure 1b). In some cases we also consider smooth functions f and g.

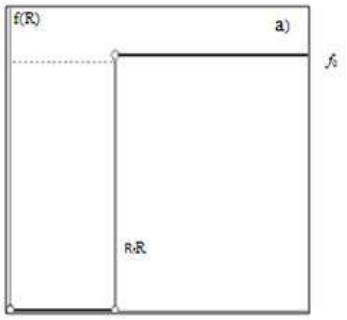

Fig.1.Function f and g.

These assumptions are consistent with plant morphogenesis. It is well known, for example, that auxin, produced in the apex, stimulates mitosis and cell proliferation. Kinetin is also known to stimulate cell proliferation. Production of mitosis factors can be self-accelerating. "The ability of M-Cdk to activate its own activator (Cdc25) and inhibit its own inhibitor (Wee1) suggests that M-Cdk activation in mitosis involves a positive feedback loop … similar molecular switches operate at various points in the cell cycle to ensure that events such as entry into mitosis occur in an all-or-none fashion." [13], [14].

3.2. Stationary solutions

In this section we study stationary solutions of the model described in the previous section. Since $f(R) = 0$ in this case, we obtain from (2.2)

$$C(x) = 1 - \frac{1 - C(L)}{L} x.$$

Then from (2.3) and (2.4)

$$C(L) = 1 - \frac{\partial L}{d} R.$$

Finally from (2.4)

$$\frac{\alpha R}{1 - \frac{\alpha}{d} L R} = g(R). \quad (2.5)$$

This equation should be completed by the condition

$$R < R_f \quad (2.6)$$

Such that $L^1(t) = 0$.

We assume in what follows that $\partial < g^1(0)$. Then for all L sufficiently large, there exists a solution R of equation (2.5) with condition (2.6). Depending on the function $g(R)$ there can exist more than one solution with the same value of L.

Denote by F(R) the left-hand side in (2.5). The standard linear stability analysis shows that the stationary solution is stable if $F^1(R) > g^1(R)$ for a solution R.

4. NUMERICAL SIMULATIONS

The functions f and g are characterized by two critical

Values: the length L (t) increases if $R>R_f$,and the production of R is strongly accelerated if R>R g.The behaviour of the system is different in two cases, $R_f > R_g$ and $R_f < R_g$.

All simulations are carried out for d = 0.001 $_{su}^2$/ tu, ó =0.009 su/tu. Here,su is a space unit. We will vary h and the initial length L_o.

4.1. Linear growth

If $R_f > R_g$, then length increase is close to a linear function of time. It reaches its stationary value, and then does not change (Figure 2). The final length depends on h and L_o = 0.1 and h from 0.001 to 0.05, L_f changes from 2.54 to 2.50. If h = 2.50. If h = 0.001 and L_o increases from 0.05 to 0.5, the final length decreases from L_f= 2.56 to L_f=2.40.

The value of R_g assumed in the simulations is 0.01. The concentration R is monotonically decreasing over time, approaching its final value $\approx$ 0.05. Therefore, the results of the simulation remain the same if the function g is identically constant (g ≡ 0.01).

4.2. Periodic growth

The behaviour of the solution to problem (2.1)-(2.4) is different if $R_f < R_g$. In this case the case the growth is periodic is periodic in time (Figure 3a). Short periods of growth are are separated by long time intervals where the long time intervals where the length does not change. The length is approximately the same during all periods of growth except for the first one, where it is one, where it is

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about two times greater. The periods without growth become longer over time. This is related to the growing length of the interval.

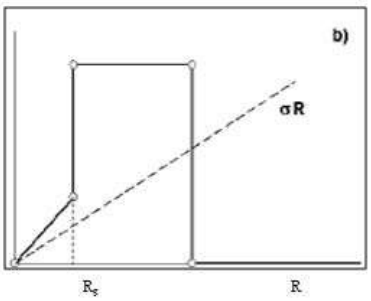

Fig.2. Linear growth

For larger L it takes more time for the concentration C (L, t) to become large enough for R (t) to increase. Figure 3b shows the function R (t).

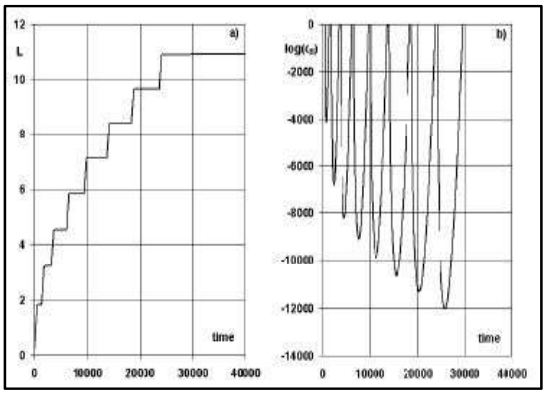

Fig.3. Perioidic growth

Contrary to the previous case, the final length L_f is very sensitive to the value of h (Figure 4). For h = 0.001 L_f = 14.56, for h = 0.003. L_f = 4.19. The number of periods of growth also varies with h. If for two different values of h the number of periods of growth is the same, then the final length depends on h weakly.

The dependence of the final length on the initial length remains weak. For h = 0.003, as L_o changes from 0.1 to 1.0 L_f changes from 4.19 to 4.61.

We recall that the first boundary condition in (2.3) determines the amount of nutrient available for the plant. The value of the concentration at the left endpoint influences the number of growth intervals and the final plant length. If we decrease the boundary condition, the length also decreases (Figure 4)

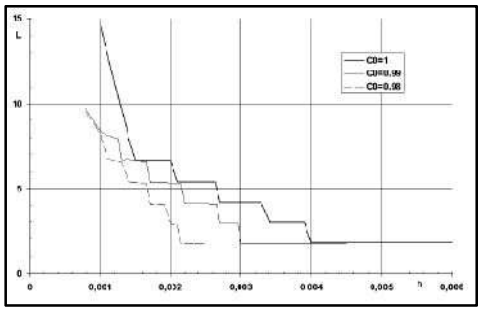

Fig.4. Dependence of the final length on *h* for different c_o

Conclusion

Genetic engineers might realize the requirement of derivatives in making accurate calculation of number of cells that occur in somatic cell that is brought into use to manufacture a carbon-copy of the living body human or non- human of which very cell has been taken. The growth rate of somatic cell can very accurately be estimated by making of derivatives. Bio-technology makes use of derivational calculus to obtain accurate and precise number value. Rate mutation, reproduction of nucleotides in cell division and their estimate value can be obtained in precise and accurate numerical value by using derivatives. Derivatives are brought in to use to measure different sorts of biological phenomenon what occur in nature with to different extremes what we call maxima and minima. Exact calculation of these two extremities can be made by using derivational equations. Such types of derivational equation enable them to determine the exact measurement and evaluation of the speed which the very process supposed to have occurred. Cytological processes can also exhibit the variations in the number of organelles. The speed with which reproduction and population enhancement has occurred can very exactly be measured by using derivatives.

The growing part of the plant, or apex, contains a narrow exterior part, the meristem where cells proliferate providing the plant growth. This layer has a constant width and consists of an approximately constant number of cell layers specific to each plant. Since it is very small compared to the whole plant, it will be considered as a mathematical surface. The displacement of this surface corresponds to the plant growth. The appearance of new cells implies that old cells exit this external layer after some time and become a part of the internal plant tissue. They differentiate, that is, they change their functions. They cannot divide any more, and they serve to conduct nutrients to the meristem.

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Vol. 3, No. 1, March-2018

REFERENCE

[1]. D'Arcy Thompson. On growth and forms. The complete revised edition. Dover, NewYork, 1992.
[2]. R. V. Jean. Phyllo taxis. A systematic study in plant morphogenesis. Cambridge University press, New York, 1994.
[3]. J. Murray. Mathematical biology. Introduction 3rded, Springer, 2001, 575p.
[4]. H.Meinhardt the algorithmic beauty of sea shells. Springer, 2003.
[5]. .L. Wolpert. Principles of development. Second Edition. Oxford University Press, Oxford, 2002.
[6]. K.J. Niklas. Computer simulations of branching-patterns and their implications on the evolution of plant. In: Some mathematical questions in biology – Plant biology. Lectures on mathematics in the life sciences, Volume 18, 1986, pp. 1-50. AMS, Providence.
[7]. C. Godin, Y. Caraglio. A multiscale model of plant topological atructures. J. Theor.Biol.191 (1998), 1-46.
[8]. F. Boudon, p. Prusinkiewicz, P. Federl, C.Godin, R. Karwowski. Interactive designof bonsai tree models. Eurographics, 22 (2003), No.3.
[9]. C. Godin et. al. 4th International workshop on functional-structural plant models. Publication UMR AMAP, 2004.