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Bi-parabolic temperature variation of elliptical plate with 2D thickness variation

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Abstract- Temperature variations are one of the most important causes of failure mechanisms in typical aerospace structures. These structures are subjected to severe thermal environments: high temperatures, high gradients and cyclic temperature changes. Due to these implications, the effects of both high-temperature and mechanical loadings have to be considered in the design process of such structures. Because the factors that affect the performance of an isolator are numerous, the type of vibration isolator to be used depends largely on the application of the device. So, it is essentially required to have the knowledge of vibration for a designer. Here, present investigation is to study the vibrations of elliptic plate with bi-dimensional parabolic varying thickness and temperature. Rayleigh-Ritz's method has been applied to derive the frequency equation of the plate. All results are illustrated with Graphs.

Index Terms- plate, thickness, frequency, taper constant

1. INTRODUCTION

For decades, engineers and scientists have endeavored to develop effective methods for mitigating unwanted vibrations. Suppressing vibrations is often a critical issue, as vibrations have been known to cause structural damage and annoyance in a whole host of systems. Such examples range from a "bumpy" ride in an automobile to fatigue failures in aircraft components. In order to minimize the effects of these unwanted vibrations, a multitude of isolation devices have been designed for use in applications ranging from machinery and automobiles to buildings and aerospace structures .Because the factors that affect the performance of an isolator are numerous, the type of vibration isolator to be used depends largely on the application of the device. In the aeronautical field, analysis of thermally induced vibrations in elliptic plate of variable thickness has a great interest due to their utility in aircraft wings. So, it is essentially required to have the knowledge of vibration for a designer.

Here, present investigation is to study the vibrations of elliptic plate with bi-dimensional parabolic varying thickness and thermal effect. Rayleigh-Ritz's method has been applied to derive the frequency equation of the plate. All results are illustrated with Graphs.

2. MODELING

It is assumed that the elliptic plate has a steady two dimensional parabolic temperature distribution given by:

$$\psi = \psi_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) \qquad (1)$$

Where ψ signifies the excess of temperature at any point on plate higher than the reference temperature and ψ_0 denotes the temperature at any point on the boundary of plate .For most of engineering materials, the temperature dependence of the modulus of elasticity is given as

$$E = E_0(1 - \gamma\psi) \qquad (2)$$

Where, $E_0$is the value of the Young's modulus at reference temperature i.e. $\psi = 0$ and γ is the slope of the variation of E with ψ. The modulus variation becomes:

$$E = E_0\left[`1 - \alpha\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\right] \qquad (3)$$

Where $\alpha = \gamma\,\psi_0(0 \le \alpha < 1)$ and α is length of a side of elliptical plate.

It is assumed that thickness χ varies parabolic in two directions as shown below:

$$\chi = \chi_0\left(1 - \beta\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\right) \qquad (4)$$

Where α is length of a side of elliptical plate and β is taper parameters in *x*- & *y*- directions respectively and $\chi = \chi_0$ at *x*=*y*=0.

Major headings should be typeset in boldface with the words uppercase.

3. SOLUTION OF PLATE MODELING

Deflection function $W(x, y)$ of plate is assumed to be finite sum of characteristics functions is taken as:

$$W(x, y) = \phi_1 + \phi_2 \qquad (5)$$

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where $\phi_1 = \left[A_2 * \left(1 - \left(1 - \frac{x}{b}\right) - \left(1 - \frac{y}{b}\right)\right)^2\right]$and

$\phi_2 = \left[A_2 * \left(1 - \left(1 - \frac{x}{a}\right) - \left(1 - \frac{y}{b}\right)\right)^3\right]$

Also, satisfying relevant geometrical boundary conditions.

Since, the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$\left.\begin{array}{ll} W = W_{,x} = 0, & x = 0, a \\ W = W_{,y} = 0, & y = 0, a \end{array}\right\} \quad (6)$$

Now assuming the non-dimensional variables as:

$$X = \frac{x}{a}, Y = \frac{y}{a}, \bar{W} = \frac{W}{a}, \bar{h} = \frac{h}{a}$$

According to Rayleigh-Ritz method, maximum strain energy and maximum kinetic energy are equal as:

$$\delta(S^* - K^*) = 0 \quad (7)$$

The kinetic energy K^* and strain energy $S^*$are:

$$K^* = \int_0^1 \int_0^{\sqrt{1-\xi^2}} \chi \, \bar{W} dY dX \quad (8)$$

and

$$S^* = \int_0^1 \int_0^{\sqrt{1-\xi^2}} D_1 \left\{\left(\bar{W}_{,xx}\right)^2 + \left(\bar{W}_{,yy}\right)^2 + 2\boldsymbol{\nu}\bar{W}_{,xx}\bar{W}_{,yy} + 2(1-\boldsymbol{\nu})\left(\bar{W}_{,xy}\right)^2\right\} dY dX \quad (9)$$

where $\xi = x/a$, D_1 is flexural rigidity and

$$D_1 = \frac{E\chi^3}{12(1-\nu^2)}.$$

By using the value of E and χ , the value of D_1 becomes:

$$D_1 = \left[E_0 \left\{1 - \alpha\left(1 - X^2 - Y^2 * \left(\frac{a}{b}\right)^2\right)\right\} * {\chi_0}^3 \left(1 - \beta\left(1 - X^2 - Y^2 * \frac{a}{b}\right)\right)^3\right] / 12(1-\nu^2) \quad (10)$$

Using equations (8) & (9) in equation (7) after putting the values of χ and D_1, one get:

$$(S^* - \lambda^2 p^2 K^*) = 0 \quad (11)$$

Where

$$S^{**} = \int_0^1 \int_0^{\sqrt{1-\xi^2}} \left[1 - \alpha\left(1 - X^2 - Y^2 * \left(\frac{a}{b}\right)^2\right)\right]\left(1 - \beta\left(1 - X^2 - Y^2 * \frac{a}{b}\right)\right)\left\{\left(\bar{W}_{,xx}\right)^2 + \left(\bar{W}_{,yy}\right)^2 + 2\boldsymbol{\nu}\bar{W}_{,xx}\bar{W}_{,yy} + 2(1-\boldsymbol{\nu})\left(\bar{W}_{,xy}\right)^2\right\} dY dX^3 \quad (12)$$

and

$$K^{**} = \int_0^1 \int_0^{\sqrt{1-\xi^2}} \left[\left(1 - \beta\left(1 - X^2 - Y^2 * \left(\frac{a}{b}\right)^2\right)\right) * \bar{W}^2\right] dY dX \quad (13)$$

Parameter of frequency is given as $\lambda^2 = \frac{12\rho a^4(1-\nu^2)}{E_0 {\chi_0}^2}$.

Now, on substituting the value of W, equation consist of two unknown constants i.e. $A_1 \& A_2$ which is evaluate as follow:

$$\frac{\partial(S^{**} - \lambda^2 K^{**})}{\partial A_n} = 0, \quad n = 1, 2 \quad (14)$$

On simplifying, we get

$$Qn_1 A_1 + Qn_2 A_2 = 0, \quad n = 1, 2 \quad (15)$$

where, Qn_1, Qn_2, $(n = 1, 2)$ include parametric constant and the frequency parameter. For having solution to be non-trivial, the determinant of the coefficient matrix should be zero. So we get equation of frequency as

$$\begin{vmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{vmatrix} = 0 \quad (16)$$

On solving equation (16), we obtained quadratic equation in λ^2. The solution value of λ^2 represent the frequency vibration of two modes i.e. λ_1 (Ist Mode) & λ_2 (IInd Mode) for clamped plate with different values of thermal gradient and taper constant.

4. RESULT AND DISCUSSION

Frequency equation (16) is quadratic in λ^2, so it will give two roots. The frequency is derived for the first two modes of vibration for homogenous elliptical plate having linearly varying thickness in both the directions, for various values of taper constant and thermal gradient. The value of Poisson ratio ν has been taken 0.345. These results are presented in figures (1-4) for first two modes of vibration for elliptical plate.

In Figure 1:- It is clearly seen that as thermal gradient increases from 0 to 1 results frequency decreases. Figure1 has shown the results for the following three cases:

β= ξ = 0.0, a/b=1.5
β= ξ = 0.3, a/b=1.5
β= ξ = 0.6, a/b=1.5

In Figure 2:- Also, it is observed that for both modes of vibration, the frequency decreases with increases in thermal gradient β from 0 to 1. Figure 2 has shown the results for the following three cases:

α = ξ=0, a/b=1.5
α= ξ =0, a/b=1.5
α= ξ =0, a/b=1.5

In Figure 3:- It is observed that for both modes of vibration, frequency parameter increases with increases in aspect ratio a/b from 0.5 to 3. Figure 3 has shown the results for the following three cases:

α =0.0, β= 0.0
α =0.3, β= 0.3
α =0.6, β= 0.6

In Figure 4:- It is seen that with increases in ξ from 0 to 1 then frequency increases for both modes of vibrations. Figure 4 has shown the results for the following three cases:

α = β=0, a/b=1.5
α= β =0, a/b=1.5
α= β =0, a/b=1.5

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Figure 1: Frequency of elliptic plate for different values of thermal gradient (α)

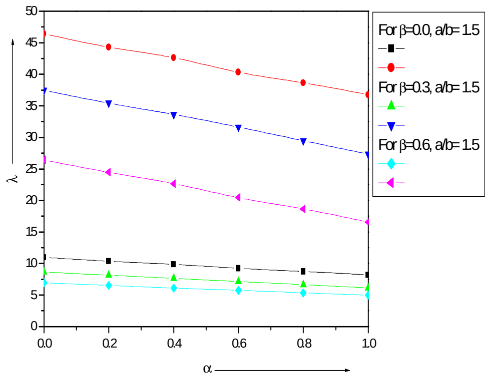


Figure 2: Frequency of elliptic plate for different values of taper constant (**β**)

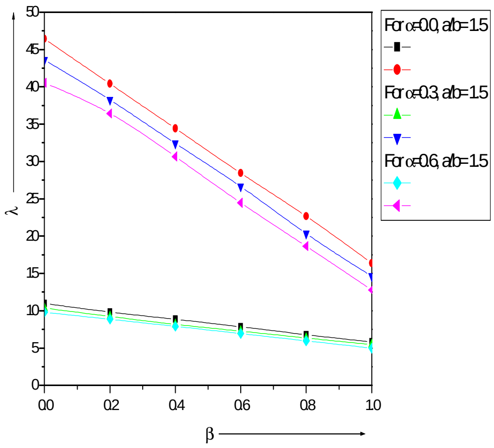


Figure 3: Frequency of elliptic plate for different values of aspect ratio (**a/b**)

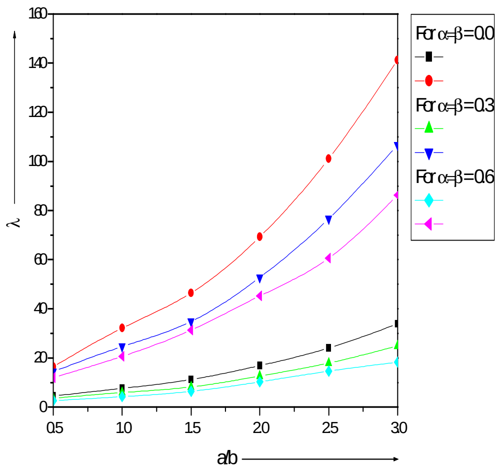
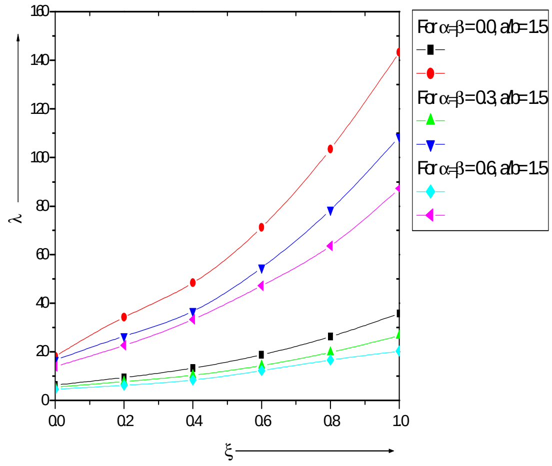


Figure 4: Frequency of elliptic plate for different values of ξ



CONCLUSION

It can be clearly seen from the figures that frequency parameter decreases with an increase in taper constant and thermal gradient. Also, frequency increases with increase in the value of aspect ratio. Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers. Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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