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Study the Effect of Parabolic Thermal Gradient and Thickness Variation on Free Vibration of Orthotropic circular Plate

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Abstract - Visco-elastic circular plate with bi-dimensional parabolic varying thickness and thermal effect are studied here. Separation of variables method is used for the differential equation has been solved for vibration of visco-elastic orthotropic circular plate. Rayleigh-Ritz technique with a two term deflection function is used to derive equation of frequency. Here, two modes of vibrations are calculated for a circular plate for different values of taper constant and thermal gradient.

Key words:-Thickness, Thermal Effect, Plate, Frequency, Taper Constant

1. INTRODUCTION

Suppressing vibrations is often a critical issue, as vibrations have been known to cause structural damage and annoyance in a whole host of systems. To minimize the effects of these unwanted vibrations, a multitude of isolation devices have been designed for use in applications ranging from machinery and automobiles to buildings and aerospace structures .Because the factors that affect the performance of an isolator are numerous, the type of vibration isolator to be used depends largely on the application of the device. In the aeronautical field, analysis of thermally induced vibrations in circular plates of variable thickness has a great interest due to their utility in aircraft wings. So, it is essentially required to have the knowledge of vibration for a designer.

Recently, Leissa [1] has given the solution for rectangular plate of variable thickness. N. Bhardwaj, A.P Gupta, K.K Choong, C.M Wang & Hiroshi Ohmori [2] discussed about transverse vibrations of clamped and simply-supported circular plates with two dimensional thickness variations. Anukul De and D. Debnath [3] studied about vibration of orthotropic circular plate withthermal effect in exponential thickness and quadratic temperature Distribution. Khanna, A., Kaur, N., & Sharma, A. K. [4] have discussed effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate. Sharma, S. K., & Sharma, A. K. [5] have discussed the mechanical vibration of orthotropic rectangular plate with 2d linearly varying thickness and thermal Effect. Khanna, A., & Sharma, A. K. [6] have solved the problem on vibration analysis of visco-elastic square plate of variable thickness with thermal gradient. Kumar Sharma, A., & Sharma, S. K. [7] have discussed the vibration computational of visco-elastic plate with sinusoidal thickness variation and linearly thermal effect in 2d. Khanna, A., Kumar, A., & Bhatia, M. [8] has investigated the computational prediction on two dimensional thermal effects on vibration of visco-elastic square plate of variable thickness. Khanna, A., & Sharma, A. K. [9] studied natural vibration of visco-elastic plate of varying thickness with thermal effect. Kumar Sharma, A., & Sharma, S. K. [10] discussed free vibration analysis of visco-elastic orthotropic rectangular plate with bi-parabolic thermal effect and bi-linear thickness variation. Sharma, S. K. & Sharma, A. K [11] discussed effect of bi-parabolic thermal and thickness variation on vibration of visco-elastic orthotropic rectangular plate. Khanna, A., & Sharma, A. K. [12] analyzed a computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect. Khanna, A., & Sharma, A. K. [13] discussed effect of thermal gradient on vibration of visco-elastic plate with thickness variation. Khanna, A., & Sharma, A. K [14] have studies the mechanical vibration of visco-elastic plate with thickness variation. Khanna, A., Kaur, N., & Sharma, A. K. [15] discussed about effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate.

Here, present investigation is to study the vibrations of circular plate with bi-dimensional parabolic varying thickness and thermal effect. Rayleigh-Ritz's method used to derive the frequency equation of the plate. All results are illustrated with Graphs.

2. EQUATION OF TRANSVERSE MOTION AND ITS SOLUTION

The equation of motion for a circular plate of radius a is governed by the equation [3]

$$r\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial}{\partial r}(rM_r)-M_\theta\right)\right]=\rho h\frac{\partial^2\omega}{\partial t^2} \qquad (1)$$

The resultant moments of M_r and M_θ for a polar visco-elastic material of plate are

$$\left.\begin{aligned} M_r &= -\widetilde{D}\ D_r\left(\frac{\partial^2\omega}{\partial r^2}+\frac{v}{r}\frac{\partial\omega}{\partial r}\right)\\ M_\theta &= -\widetilde{D}D_\theta\left(\frac{1}{r}\frac{\partial\omega}{\partial r}+v\frac{\partial^2\omega}{\partial r^2}\right)\end{aligned}\right\} \quad (2)$$

where

$$D_r = \frac{E_r h^3}{12(1-v_\theta v_r)} \text{ and } D_r = \frac{E_\theta h^3}{12(1-v_\theta v_r)} \quad (3)$$

and $\widetilde{D}$is the visco-elastic operator.

The deflection ω can be sought in the form of product of two functions as follows:

$$w(r,\theta,t) = W(r,\theta)T(t) \quad (4)$$

where $W(r,\theta)$ is the deflection function and $T(t)$ is the time function. Using equations (2) and (4) in (1) one gets

$$D_r\frac{\partial^4 w}{\partial r^4}+\frac{2}{r}\left(D_r+r\frac{\partial D_r}{\partial r}\right)\frac{\partial^3 w}{\partial r^3}+\frac{1}{r^2}\left[-\overline{D_\theta}+r(2+v_\theta)\frac{\partial D_r}{\partial r}+r^2\frac{\partial^2 D_r}{\partial r^2}\right]\frac{\partial^2 w}{\partial r^2}+\frac{1}{r^3}\left[\overline{D_\theta}+r\frac{\partial\overline{D_\theta}}{\partial r}+r^2\frac{\partial^2 D_r}{\partial r^2}\right]\frac{\partial w}{\partial r}+\bar{\rho}h\frac{\partial^2 w}{\partial t^2}=0 \quad (5)$$

Substituting dimensionless quantities,

$$R=\frac{r}{a}, H=\frac{h}{a}, \rho=\frac{\bar{\rho}}{a}, D_R=\frac{D_r}{a^3}, D_\theta=\frac{\overline{D_\theta}}{a^3}$$

and $\bar{W}=\frac{w}{a}$

These equations are expressions for transverse motion of a circular plate with variable thickness.

Equation (1) will be transformed in to the following form [3]

$$D_R\frac{\partial^4\bar{W}}{\partial R^4}+\frac{2}{r}\left(D_R+R\frac{\partial D_R}{\partial R}\right)\frac{\partial^3\bar{W}}{\partial R^3}+\frac{1}{R^2}\left[-D_\theta+R(2+v_\theta)\frac{\partial D_R}{\partial R}+r^2\frac{\partial^2 D_R}{\partial R^2}\right]\frac{\partial^2\bar{W}}{\partial R^2}+\frac{1}{R^3}\left[D_\theta+R\frac{\partial D_\theta}{\partial R}+R^2\frac{\partial^2 D_R}{\partial R^2}\right]\frac{\partial\bar{W}}{\partial R}+a^3\rho H\frac{\partial^2\bar{W}}{\partial t^2}=0$$

Consider, temperature varies parabolic in the radial and circumference directions for a circular plate as [2]

$$T = T_0(1-R^2)(1-\cos^2\theta) \quad (6)$$

where$T$ denotes the temperature excess above the reference temperature and T_0 denotes the reference temperature.

The temperature dependence of the modulus of elasticity for most structural material is given as

$$E_R(T) = E_1(1-\gamma T), E_\theta(T) = E_2(1-\gamma T) \quad (7)$$

where E_0 is the value of Young's modulus at the reference temperature, i.e. T =0 and γ is the slope of variation $E$of with$T$. The module variation, in view of expressions (6) and (7), becomes

$$E_R(r) = E_1[1-\alpha(1-R^2)(1-\cos^2\theta)], E_\theta(r) = E_2[1-\alpha(1-R^2)(1-\cos^2\theta)] \quad (8)$$

where $\alpha=\gamma T_0$ $(0\le\alpha<1)$ is a parameter known as thermal gradient.

The expression for the maximum strain energy V and maximum kinetic energy T in the plate, when it vibrates with the mode shape $W(r,\theta)$ are given as [3]

$$V=\frac{1}{2}\int_0^a\int_0^{2\pi}\left[D_R\left\{\left(\frac{\partial^2 W}{\partial R^2}\right)+2v_\theta\frac{\partial^2 W}{\partial R^2}\left(\frac{1}{R}\frac{\partial W}{\partial R}\right)\right\}+D_\theta\left(\frac{1}{R}\frac{\partial W}{\partial R}\right)^2\right]R\,dR\,d\theta \quad (9)$$

and

$$T=\frac{1}{2}p^2\int_0^a\int_0^{2\pi}\rho H\,W^2 R\,dR\,d\theta \quad (10)$$

It is assumed that the thickness varies in parabolic in two dimensional as

$$H = H_o F^2(R,\theta) \quad (11)$$

where $(H_0 = H|_{\cdot R=0})$and $F(R,\theta)=(1-\beta_1 R^2)(1-\beta_2 Cos\,^2\theta)$

Assume that

$$W(r,\theta) = W_1(r)cos\theta \quad (12)$$

Using equations (6) and (11) in equations (9) and (10), we get

$$V=\frac{\pi a^3 E_0 {H^3}_0}{24(1-v^2)}\int_0^1(1-\alpha(1-R^2)(1-\cos^2\theta))\left((1-\beta_1R^2)(1-\beta_2 Cos\,^2\theta)\right)^3\left\{\left[\left(\frac{d^2\bar{W}}{dR^2}\right)+2v_\theta\frac{d^2\bar{W}}{dR^2}\left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)+\left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)^2\right]\right\}RdR \quad (13)$$

and

$$T=\frac{\pi a^8 p^2\rho H_0}{2}\int_0^1[(1-\beta_1R^2)(1-\beta_2 Cos\,^2\theta)]R\bar{W}^2 dR \quad (14)$$

Here, Rayleigh-ritz technique requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that

$$\delta(V-T)=0 \quad (15)$$

For arbitrary variation of W satisfying relevant geometric boundary conditions. Circular plate clamped at the edges r=ai.e. R=1 the boundary conditions are

$$\bar{W}=\frac{d\bar{W}}{dR}=0 \quad \text{at} R=1 \quad (16)$$

and the corresponding two terms of deflection function is taken as

$$\bar{W}(R) = C_1(1-R^2)+C_2(1-R)^3 \quad (17)$$

where $C_1$and $C_2$are undetermined coefficients.

Now using equations (13) and (14) in equation (15), we get

$$\delta(V_1-\lambda^2 T_1)=0 \quad (18)$$

Where

$$V_1=\int_0^1(1-\alpha(1-R^2)(1-\cos^2\theta))\left((1-\beta_1R^2)(1-\beta_2 Cos\,^2\theta)\right)^3\left\{\begin{bmatrix}\left(\frac{d^2\bar{W}}{dR^2}\right)+2v_\theta\frac{d^2\bar{W}}{dR^2}\\ \left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)+\left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)^2\end{bmatrix}\right\}RdR \quad (19)$$

and

$$T_1=\int_0^1[(1-\beta_1R^2)(1-\beta_2 Cos\,^2\theta)]R\bar{W}^2 \quad (20)$$

where

$$l=\frac{\pi a^5 E_0 {H_0}^3}{24(1-v^2)} \quad (21)$$

Equation (17) involves the unknowns C_1 and C_2 arising due to substitution of $\overline{W}(R)$ from (16). These unknowns are to be determined from equation (18), for which

—— (22)

Equation (16) simplifies to the form

(23)

Where (involve the parametric constant and frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (20) must be zero. Thus, one gets the frequency equation as

(24)

On solving (24) one gets a quadratic equation in λ^2, so it will give two roots.

3. RESULT AND DISCUSSION

Here, frequencies for the first two modes of vibrations are computed for circular plate whose thickness varies parabolic in two directions for different values of thermal gradient α and taper constants β_1 and β_2, has been considered. All results are presented in graphical form.

Table 1 contains numerical results for frequency parameter λ for different values of thermal gradient α form 0.0 to 1 and taper constants

a) $\beta_1 = \beta_2 = 0.0$
b) $\beta_1 = \beta_2 = 0.4$
c) $\beta_1 = \beta_2 = 0.8$

It can be seen from the tables that as thermal gradient α increases, frequency parameter decreases for both the modes of vibration.

Table 2 contains numerical results for frequency parameter λ for different values of thermal gradient α form 0.0 to 1 and taper constants

a) $\beta_1 = 0.0$, $\beta_2 = 0.2$
b) $\beta_1 = 0.2$, $\beta_2 = 0.4$
c) $\beta_1 = 0.4$, $\beta_2 = 0.8$

It can be seen from the tables that as thermal gradient α increases, frequency parameter also decreases for both the modes of vibration.

Table 3 contains numerical results for frequency parameter λ for different values of thermal gradient β_1 form 0.0 to 1 and taper constants

a) $\alpha = \beta_2 = 0.0$
b) $\alpha = \beta_2 = 0.4$
c) $\alpha = \beta_2 = 0.8$

It can be seen from the tables that as thermal gradient β_1 increases, frequency parameter increases for both the modes of vibration.

Table 4 contains numerical results for frequency parameter λ for different values of thermal gradient β_2 form 0.0 to 1 and taper constants

a) $\alpha = \beta_1 = 0.0$
b) $\alpha = \beta_1 = 0.4$
c) $\alpha = \beta_1 = 0.8$

It can be seen from the tables that as thermal gradient β_1 increases, frequency parameter also increases for both the modes of vibration.

Table 1:- Frequency parameter λ for different values of thermal gradient

α	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = \beta_2 = 0.4$		$\beta_1 = \beta_2 = 0.8$	
0.0	18.66	57.82	25.54	55.52	15.42	51.24
0.2	15.82	54.83	22.64	52.51	13.46	48.23
0.4	13.61	51.86	19.55	49.52	11.48	46.25
0.6	11.59	48.86	16.67	46.54	8.47	43.34
0.8	9.62	44.84	13.62	43.49	6.62	41.30
1	7.66	41.86	10.59	40.52	3.53	38.29

Figure 1:- Frequency parameter vs thermal gradient

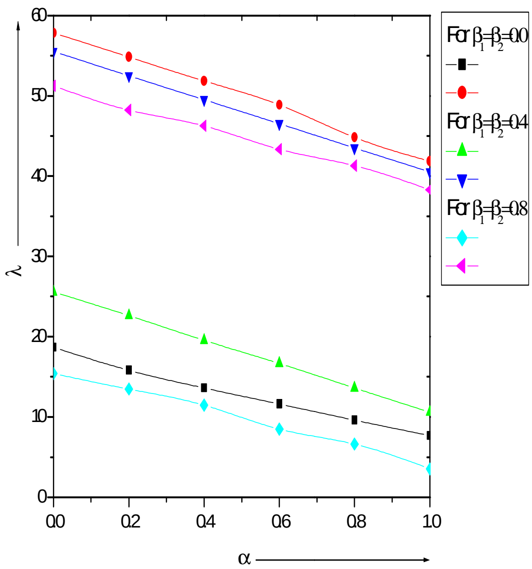


Table 2:- Frequency parameter λ for different values of thermal gradient

α	$\beta_1 = 0.0$, $\beta_2 = 0.2$		$\beta_1 = 0.2$, $\beta_2 = 0.4$		$\beta_1 = 0.4$, $\beta_2 = 0.8$	
0.0	25.62	68.23	22.42	60.44	18.52	55.74
0.2	24.59	64.33	21.38	57.48	17.53	52.72
0.4	23.53	60.38	20.44	54.43	16.56	49.76
0.6	22.66	56.36	19.45	51.46	15.51	47.73
0.8	21.54	52.34	18.39	48.44	14.54	45.71
1	20.63	48.30	17.45	45.43	13.55	43.77

Figure 2:- Frequency parameter vs thermal gradient

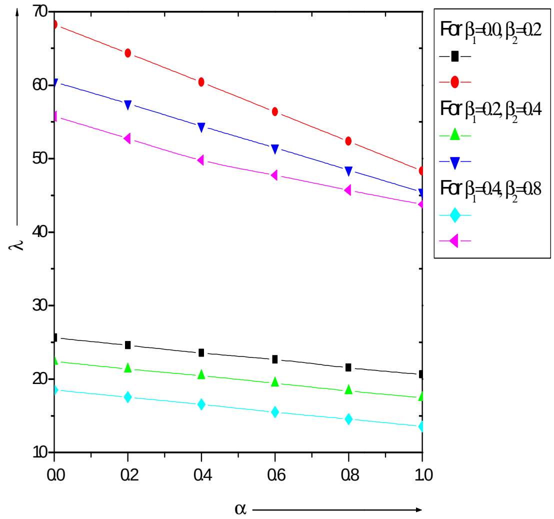


Table 3:- Frequency parameter λ for different values of taper constant β_1 form 0.0 to 1

β_1	$\beta_2=\alpha=0.0$		$\beta_2=\alpha=0.4$		$\beta_2=\alpha=0.8$	
0.0	25.34	66.47	21.66	60.34	18.55	56.72
0.2	28.38	70.42	23.63	64.33	20.59	60.68
0.4	31.37	74.43	25.67	68.32	22.56	64.73
0.6	34.34	78.48	27.62	72.39	24.54	68.64
0.8	37.38	82.44	29.65	76.34	26.53	72.68
1	40.32	86.47	31.62	80.35	28.66	74.69

Figure 3:- Frequency parameter vs taper constant β_1

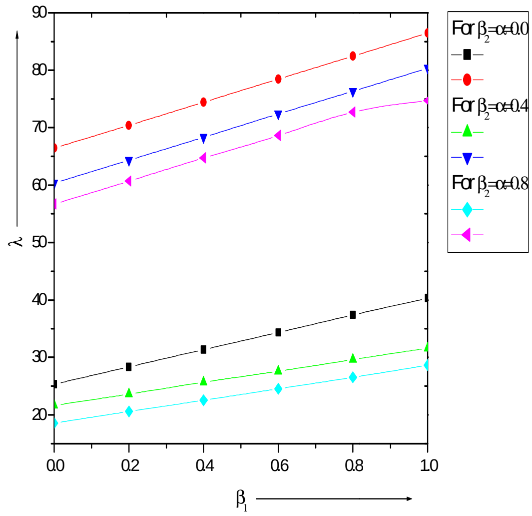
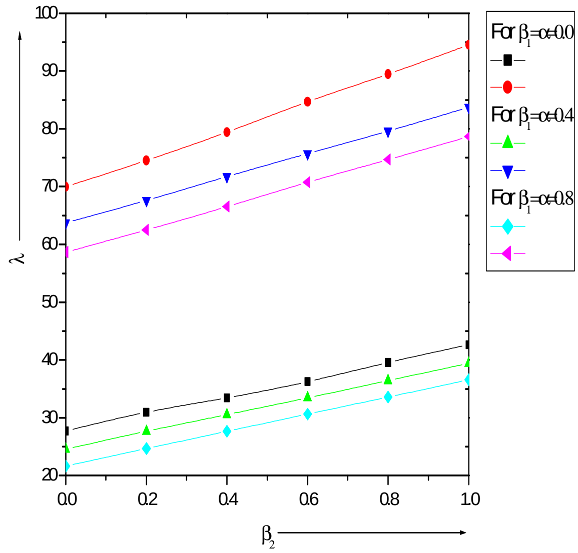


Table 4:- Frequency parameter λ for different values of taper constant β_2 form 0.0 to 1

β_2	$\beta_1=\alpha=0$		$\beta_1=\alpha=0.4$		$\beta_1=\alpha=0.8$	
0.0	27.68	69.96	24.56	63.71	21.64	58.65
0.2	30.93	74.54	27.65	67.58	24.66	62.47
0.4	33.43	79.43	30.53	71.73	27.64	66.52
0.6	36.25	84.67	33.42	75.69	30.63	70.73
0.8	39.54	89.44	36.42	79.58	33.59	74.68
1	42.64	94.53	39.43	83.71	36.58	78.63

Figure 4:- Frequency parameter vs taper constant β_2



4. CONCLUSION

Present, study devoted to the effect of material on the fundamental frequencies of circular plate whose thickness and thermal effect varies parabolic in two directions based on classical plate theory. Different values of taper constant and thermal gradient have been considered. It is observed on increasing the value of thermal gradient, frequency parameter decreases where as on increasing thermal gradient the value of frequency parameter increases. Thus engineers can obtained a change in the frequencies of a plate by a proper choice of various plate parameters considered here and fulfill their requirements.

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