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Transportation Problems: A Special case for Kalka Based flour Mill

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Abstract-. Transportation problem is one of the predominant areas of operations research, widely used as a decision making tool in engineering, business management and many other field. The main objective of this dissertation is to select an appropriate choice from the multi-choices for the cost coefficients of the objective function and the demand of the constraints in the TP by assigning the value in such a way that the total cost is minimized and satisfies the required demand.

In this we have solved the transportation problem of the flour Mill situated at Kalka. The mill has to deliver the flour to three destinations namely Manpura, Baddi and Nanakpur. We have discussed the transportation problem using two methods namely Vogel Approximation Method and the Stepping-Stone Method. Out of these two Methods the stepping-stone Method is better as it provides the least transportation cost as compared to the Vogel Approximation Method

Keywords- Transportation problem,\Linear programming, Operation research, stepping stone ,Vogel approximation

1. LINEAR PROGRAMMING

Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtained under those conditions. A typical example would be taking the limitation of materials and labor, and then determining the "best" production levels for maximal profits under those conditions.

In "real life", linear programming is part of very important area of mathematics called "optimization techniques". This field of study is used every day in the organization and allocation of resources. These "real life" systems can have dozens or hundreds of variable, or more. In algebra, though, you'll only work with the simple two-variable linear case.

The general process for solving linear-programming exercise is to graph the inequalities (called the "constraint")to form a walled-off area on the x, y-plane(called the "feasibility region").Then you figure out the coordinate of the corners of this feasibility region(that is, you find the intersection points of the various pair of lines),and these corner points in the formula(called the "optimization equation") for you' trying to find the highest or lowest value.

Linear program are problems that can be expressed in canonical form as

Maximize $A^T x$

Subject to $Bx \leq c$

And $x \geq 0$

Where x represents the vector of variable (to be determined), c and b are vectors of (known coefficients, A is a (known) matrix of called the function (cut in the case). The inequalities $Bx \leq c$ and $x \geq 0$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions. If every entry in the first is less- than or equal- to the corresponding entry in the second then we can say the first vector is less- than or equal-to the second vector

1.1 Application

- **Personnel Management**:

LP technique enables the personnel manager to solve problems relating to recruitment, selection, training, and deployment of manpower to different departments of the firm. It is also used to determine the minimum number of employees required in various shifts to meet production schedule within a time schedule.

- **Inventory Management:**

A firm is faced with the problem of inventory management of raw materials and finished products. The objective function in inventory management is to minimise inventory cost and the constraints are space and demand for the product. LP technique is used to solve this problem.

- Marketing Management:

LP technique enables the marketing manager in analysing the audience coverage of advertising based on the available media, given the advertising budget as the constraint. It also helps the sales executive of a firm in finding the shortest route for his tour. With its use, the marketing manager determines the optimal distribution schedule for transporting the product from different warehouses to various market locations in such a manner that the total transport cost is the minimum.

- . Blending Problem:

LP technique is also applicable to blending problem when a final product is produced by mixing a variety of raw materials. The blending problems arise in animal feed, diet problems, petroleum products, chemical products, etc. In all such cases, with raw materials and other inputs as constraints, the objective function is to minimise the cost of final product.

- **Telecommunication**

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Another application of linear algebra lies in telecommunications. If there ara telephone calls being transmitted across a multipoint phone line network, linear programming provides a technique to find where it is necessary to build extra capacity.

1.2 Advantages and limitations:

LP has been considered an important tool due to following reasons:

i). LP makes logical thinking and provides better insight into business problems.
ii). Manager can select the best solution with the help of LP by evaluating the cost and profit of various alternatives.
iii). LP provides an information base for optimum allocation of scarce resources.
iv). LP assists in making adjustments according to changing conditions.
v). LP helps in solving multi-dimensional problems.

- **LP approach suffers from the following limitations also**:

i). This technique could not solve the problems in which variables cannot be stated quantitatively.
ii). In some cases, the results of LP give a confusing and misleading picture. For example, the result of this technique is for the purchase of 1.6 machines.It is very difficult to decide whether to purchase one or two-machine because machine can be purchased in whole.
iii). LP technique cannot solve the business problems of non-linear nature.
iv). The factor of uncertainty is not considered in this technique.
v). This technique is highly mathematical and complicated.
vi). If the numbers of variables or constrains involved in LP problems are quite large, and then using costly electronic computers become essential, which can be operated, only by trained personel.
vii). Under this technique to explain clearly the objective function is difficult.

2. TRANSPORTATION PROBLEM

The transportation problem is a special type of linear programming where the objective is to minimise the cost of distributing a product from a number of sources or origin to a number of destinations .Because of its special structure the usual simplex method is not suitable for solving transportation problem. These problems require a special method of solution. The origin of a transportation problem is the location from which shipments are dispatched. The destination of a transportation problem is the location to which shipments are transported. The unit transportation cost is of transporting cost is the cost of transporting one unit of the consignment from an origin to a destination. To achieve this objective we must know the amount and location of available supplies and the quantities demanded in addition to the involvement of cost associated with each “sending”. It helps in solving problems on distribution and transportation of resources from one place to another.

The goods are transported from a set of sources (e.g. factory) to a set of destination (e.g. warehouse) to meet the specific requirements.

The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouse or distribution centers.

Suppose there are more than one centers, called ‘destinations’ and cost of transporting or shipping from each of the origin to each of the destination being different and known. The problem is to transport the goods from various origins to different destinations in such manner that the cost of shipping or transportation is minimum.

Thus, the transportation problem is to transport various amounts of a single homogenous commodity, which are initially stored at various origins, to different destinations in such a way that the transportation cost is minimum.

Business and Industries are practically faced with both economic optimizations such as cost minimization of non-economic items that are vital to the existence of their firms. The transportation models or problems are primarily concerned with optimal (best possible) way in which product produced at different factories or plants (called supply origins) can be transported to a number of warehouse or customers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.) there is a need to minimize the cost of transportation so as to increase profit on sales.

The transportation problem is a special class of linear programming problem, which deals with shipping commodities from source to destinations. The objective of the transportation problem is to determine the shipping schedule that minimize that total shipping cost while satisfying supply and demands limits.

The transportation problem has an application in industry, communication network, planning, scheduling transportation and allotment etc.

Considers a situation in which three points of origin in (A_1, A_2, A_3) have suppliers available to meet needs at three destinations (N_1, N_2, N_3). The amount available at each is specified (a_1, a_2, a_3) as also the amounts needed at each destinations (n_1, n_2, n_3). Furthersome the cost of moving goods between origin and destination can be setup as a table of m_{ij} where the subscripts

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indicate the cell, given the cost of moving from the i'th origin to j'th destination.

Example m_{22} is the cost of moving goods from the origin A_2 to destination N_2. In a real life problem these quantities must be written in specific units: the supplies available and needs might be in tonnes, or even in thousands of tones; the movements costs will then be in cost units per tonne, for example $/tonne.

The transportation problem received this name because many of its applications involve in determining how to optimally transport goods. Transportation problem is a logistical problem for organizations especially for manufacturing and transport companies. This method is a useful tool in decision-making and process of allocating problem in these organizations. The transportation problem deals with the distribution of goods from several points, such as factories often known as sources, to a number of points of demand, such as warehouses, often known as destinations. Each source is able to supply a fixed number of units of products, usually called the capacity or availabilit**y**, and each destination has a fixed demand, usually known as requirement**.** Because of its major application in solving problems which involving several products sources and several destinations of products, this type of problem is frequently called "The Transportation Problem". The classical transportation problem is referred to as special case of Linear Programming (LP) problem and its model is applied to determine an optimal solution of delivery available amount of satisfied demand in which the total transportation cost is minimized. The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

. **2.1 Objective**

Transportation model is used in the following:

- To decide the transportation of new materials from various centers to different manufacturing plants. In the case of multi – plant company this is highly useful.
- To decide the transportation of finished goods from different manufacturing plants to the different distribution centers. For a multi-plant-market company this is useful.

These two are the uses of transportation model. The objective is to minimize transportation cost.

3.METHODOLOGY

This chapter reviews the proposed solution methodology and approach for handling transportation problem in Guinness Ghana Ltd. The transportation problem seeks to minimize the total shipping costs of transporting goods from *m* origins (each with a supply s_i) to *n* destinations (each with a demand *dj*), when the unit shipping cost from an origin, *i*, to a destination, *j*, is c_{ij}.

These two are the uses of transportation model. The objective is minimizing transportation cost This is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem**.**

In a transportation problem, we have certain origins, which may represent factories where we produced items and supply a required quantity of the products to a certain number of destinations. This must be done in such a way as to maximize the profit or minimize the cost. Thus we have the places of production as origins and the places of supply as destinations. Sometimes the origin and destinations are also termed as sources and sinks.

Transportation model is used in the following:

To decide the transportation of new materials from various centres to different manufacturing plants. In the case of multi-plant company this is highly useful.

To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful.

3.1 Mathematical Problem

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

The total supply of the products from warehouse *i* = a *i, where i* 1, **2, 3**...*m*

The total Demand of the products at the outlet *j = j b*, where *j* 1, 2, 3...*n*.

The cost of sending one unit of the product from warehouse *i* to outlet *j* is equal to

C*ij*, where *i* 1, 2, 3...*m* and *j* 1, 2, 3...*n*. The total cost of a shipment is the linear in size of shipment.

3.2 The Decision Variables

The variables in the Linear Programming(LP) model of the TP will hold the values for the number of units shipped from one source to a destination.

The decision variables are:

*Xi*j = the size of shipment from warehouse *i* to outlet *j* ,Where *i* 1, 2, 3...*m* and *j* 1, 2, 3...*n*.

This is a set of m.n variables.

3.3The Objective Function

The objective function contains costs associated with each of the variables. It is a minimization problem.

Consider the shipment from warehouse *i* to outlet *j*. For any *i* and *j*, the transportation cost per unit *ij C* and the size of the shipment is *Xij*. Since we assume that

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the total cost function is linear, the total cost of this shipment is given by $c_{ij}x_{ij}$

Summing over all i and j now yields the overall transportation cost for all warehouse-outlet combinations. That is, our objective function is:

Minimize

$${}_{ij}\, C_{ij}$$

3.4 The Constraints

The constraints are the conditions that force supply and demand needs to be satisfied. In a Transportation Problem, there is one constraint for each node.

Let **a1** denote a source capacity and **b1** denote destination needs

1) The supply at each source must be used:

$${}_{ij} = a_{i,}\ i= 1, 2, 3\ldots\ m$$

2) The demand at each destination must be met:

$${}_{ij} = a_{ij}\ ,\ j=1,2,3\ldots\ldots,n$$

And

3) No negativity:

$$X_{ij} \geq 0, \text{ for all i and j}$$

The transportation model will then become:

Minimizing the transportation cost

$$\text{Minimize } Z = \qquad {}_{ij}\, C_{ij} \ldots\ldots\ldots \quad (1)$$

$${}_{i\ j} \leq a_{i\ ,}\ (\ i=\ 1,2,3,\ldots.m\ )\ldots\ (2)$$

(Demand Constraint)

$${}_{ij} \geq b_j\ ,\ (\ j=1,2,3,\ldots\ldots n\quad)\ldots..(3)$$

(Supply Constraint)

$$X_{ij} \geq o, (\ i=1,2,3\ldots.m;\ j=1,2,3\ldots.n\ )$$

This is a linear program with $m.n$ decision variables, $m+\ n$ functional constraints, and $m.n$ nonnegative constraints.

m =Number of sources

n =Number of destinations

a_i = Capacity of i^{th} source (in tons, pounds, liters, etc)

$b_{j=}$ Demand of j^{th} destination (in tons, pounds, liters, etc.)

c_{ij}= cost coefficients of material shipping (unit shipping cost) between i *th* source

And $j^{\ th}$ destination (in $ or as a distance in kilometers, miles, etc.)

x_{ij}= amount of material shipped between $i^{\ th}$ source and $j^{\ th}$ destination (in

Tons, pounds, liters etc.)

3.5 Balanced Transportation Problem

IF

$${}_{i} = \qquad {}_{j}$$

3.6 Unbalanced Transportation Problem

If $\qquad j \neq \qquad j$

The transportation problem is known as an unbalanced transportation problem. There are two cases

Case (1)

$${}_{i}\ \square \qquad {}_{j}$$

Case (2)

$${}_{i}\ \square \qquad {}_{j}$$

Introduce a dummy origin in the transportation table; the cost associated with this origin is set equal to zero. The availability at this origin is:

$${}_{j} = \qquad {}_{i} = 0$$

3.7 Transportation Tableau

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

The model of a transportation problem can be represented in a concise tabular form with all the relevant parameters.

The transportation tableau (A typical TP is represented in standard matrix form), where supply availability (ai) at each source is shown in the far right column and the destination requirements (bi) are shown in the bottom row. Each cell represents one route. The unit shipping cost (Cij) is shown in the upper right corner of the cell; the amount of shipped material is shown in the centre of the cell. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

Table 1: THE TRANSPORTATION TABLEAU

To Destination→ ↓From Source↓	D_1	D_2	…Dj…	D_n	Source Supply
S_1	c_{11} x_{11}	c_{12} x_{12}		c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	a_2
…S_i…			c_{ij} x_{ij}		…a_i…
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	a_m
Destination Requirements	b_1	b_2	…b_j…	b_m	$\sum a_i$ $\sum b_j$

- **Degency in Transportation Problem**

Degeneracy exists in a transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one (m + n - 1). Degeneracy may be observed either during the initial allocation when the first entry in a row or column satisfies both the row and column requirements or during the Stepping stone method application, when the added and subtracted values are equal. Transportation with ***m***-origins and ***n***-destinations can have **m+n-1** positive basic variables, otherwise the basic solution degenerates. So whenever the number of basic cells is less than ***m + n-1,*** the transportation problem is degenerate. To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete ***m +n –1*** basic variable

- **The Initial Basic Feasible Solution (BFS)**

Let us consider a T.P involving m origins and n destinations. Since the sum of origin capacities equals the sum of destination requirements, a feasible

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solution always exists. Any feasible solution satisfying m + n – 1 of the m + n constraints is a redundant one and hence can be deleted. This also means that a feasible solution to a T.P can have at the most only m + n – 1 strictly positive component, otherwise the solution will degenerate. It is always possible to assign an initial feasible solution to a T.P. in such a manner that the rim requirements are satisfied. This can be achieved either by inspection or by following some simple rules. We begin by imagining that the transportation table is blank i.e. initially all xij = 0. The simplest procedures for initial allocation discussed in the following section.

- **Feasible Solution (F.S.)**

A set of non-negative allocations xij > 0 which satisfies the row and column restrictions is known as feasible solution.

- **Basic Feasible Solution (B.F.S.)**

A feasible solution to a m-origin and n-destination problem is said to be basic feasible solution if the number of positive Allocations are (m+n–1). If the number of allocations in a basic feasible solutions are less than (m+n–1), it is called degenerate basic feasible solution (DBFS) (Otherwise non-degenerate).

- **Optimal Solution**

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost

- **Cell:**

It is a small compartment in the transportation tableau

Circuit: A circuit is a sequence of cells (in the balanced transportation tableau) such that (i) It starts and ends with the same cell.

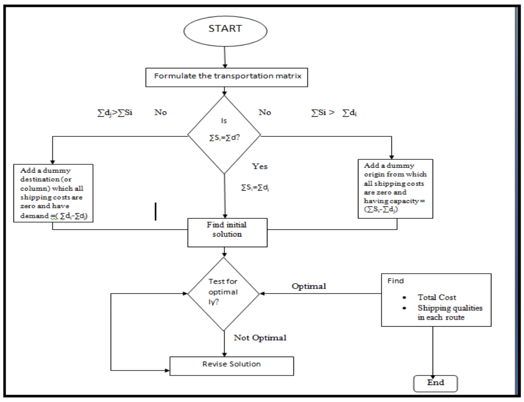
(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.

- **Allocation**: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraint

3.8 Solution for a Transportation Problem

3.10.1 Flow Chart Solution For the transportation problem



The transportation problem approach..

Summary description of the Flow chart

1) First the problem is formulated as transportation matrix.
2) Check weather is a balance transportation model?
3) If not balance add a dummy to either the supply or the demand to balance the transportation model**.**
4) Find the initial solution of the transportation problem.
5) Check whether the solution is optimized? If the solution is not optimize Go to 4.
6) When optimal solution is obtained.
7) We compute the total transportation cost and also shipped the respective quantity demand to its route.

3.9 Solution Algorithm For the transportation Problem

Transportation models do not start at the origin where all decision values are zero; they must instead be given an initial feasible solution

The solution algorithm to a transpiration problem can be summarized into following steps:

Step1. Formulate the problem and set up in the matrix form. The formulation of transportation problem is similar to LP problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination, respectively.

Step 2. Obtain an initial basic feasible solution.

This initial basic solution can be obtained by using any of the following methods

i). North West Corner Rule
ii). Matrix Minimum (Least Cost) Method
iii). Vogel Approximation Method

The solution obtained by any of the above methods must fulfill the following conditions:

- The solution must be feasible, i.e., it must satisfy all the supply and demand constraints. This is called RIM CONDITION**.**

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- The number of positive allocation must be equal to m + n – 1, where, m is number of rows and n is number of columns.

The solution that satisfies the above mentioned conditions are called a non-degenerate basic feasible solution.

Step 3. Test the initial solution for optimality. Using any of the following methods can test the optimality of obtained initial basic solution:

- Stepping Stone Method
- Modified Distribution Method (MODI) If the solution is optimal then stop, otherwise, determine a new improved solution.

Step 4. Updating the solution

Repeat Step 3 until the optimal solution is arrived at.

3.10 BASIC FEASIBLE SOLUTION OF BALANCED TRANSPORTATION PROBLEMS

1) **Vogel's Approximation Method (VAM)**

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The opportunity cost for a given supply row or demand column is defined as the difference between the lowest cost and the next lowest cost alternative. This method is preferred over the methods discussed above because it generally yields, an optimum, or close to optimum, starting solutions. Consequently, if we use the initial solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced. The steps involved in determining an initial solution using VAM are as follows:

Step1. Write the given transportation problem in tabular form (if not given).

Step2. Compute the difference between the minimum cost and the next minimum cost corresponding to each row and each column which is known as penalty cost.

Step3. Choose the maximum difference or highest penalty cost. Suppose it corresponds to the i^{th} row. Choose the cell with minimum cost in the i^{th} row. Again if the maximum corresponds to a column, choose the cell with the minimum cost in this column.

Step4. Suppose it is the (i, j) th cell. Allocate min (ai, bj) to this cell. If the min $(a_i, b_j) = a_i$, then the availability of the i^{th} origin is exhausted and demand at the j^{th} destination remains as bj-ai and the i^{th} row is deleted from the table. Again if min $(a_i, b_j) = b_j$, then demand at the jth destination is fulfilled and the availability at the i'th origin remains to be a_i-bj and the jth column is deleted from the table.

Step5. Repeat steps 2, 3, 4 with the remaining table until all origins are exhausted and all demands are fulfilled.

- Method is based on the concept of *penalty cost* or *regret.*
- A penalty cost is the difference between the largest and the next largest cell cost in a row (or column).
- In VAM the first step is to develop a penalty cost for each source and destination.
- Penalty cost is calculated by subtracting the minimum cell cost from the next higher cell cost in each row and column.
- **Vogel's Approximation Method (VAM) Summary of Steps**
- Determine the penalty cost for each row and column.
- Select the row or column with the highest penalty cost.
- Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
- Repeat steps 1, 2, and 3 until all rim requirements have been met
- **Illustrative Example on Transportation Problem**
- In this tableau the decision variable Xij , represent the number of tons of wheat Flour transported from each grain elevator, in Kalka i(where i=1,2,3) ,to each mill, j(where j= A,B,C) in Manpura,Baddi,Nanakpur . The objective function represents the total transportation cost for each route. Each term in the objective function reflects the cost of the tonnage transported for one route. The problem is to determine how many tons of wheat of transport from each grain elevator to each mill on monthly basis in order to minimize the total cost of transportation
- **A balanced transportation problem**

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

Table2: A balanced transportation problem

- **The VAM Penalty Costs**

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7	11	11	175	4
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

Table 3: The VAM Penalty Costs

- **The Initial VAM Allocation**

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VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7 175	11	11	175	
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	2		

Table 4: The Initial VAM Allocation

- **The Second VAM Allocation**

After each VAM cell allocation, all row and column penalty costs are recomputed

From \ To	A	B	C	Supply	
1	6	8	10	150	4
2	7 175	11	11	175	
3	4	5 100	12	275	8
Demand	200	100	300	600	
	2		2		

Table 5: The Second VAM Allocation

- **The Third VAM Allocation**

Recomputed penalty costs after the third allocation

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

Table 6: The Third VAM Allocation

The Initial VAM Solution

Total cost = 10(150) + 4(25) + 5(100) + 12(150) + 7(175)
= Rs.5, 125

3.13 METHODS FOR SOLVING TRANSPORTATION PROBLEMS TO OPTIMALITY

1) An Optimal Solution

To obtain an optimal solution by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. An optimal solution is one where there is no other set of transportation routes that will further reduce the total transportation cost. Thus, we have to evaluate each unoccupied cell in the transportation table in terms of an opportunity of reducing total transportation cost. An unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes (allocations). This value indicates the per unit cost reduction that can be achieved by raising the shipment allocation in the unoccupied cell from its present level of zero. This is also known as an incoming cell (or variable). The outgoing cell (or variable) in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero first as more units are allocated to the unoccupied cell with largest negative opportunity cost. That is, the current solution cannot be improved further. This is the optimal solution

The widely used methods for finding an optimal solution are:

- Stepping stone method (not to be done).
- Modified Distribution (MODI) method.

They differ in their mechanics, but will give exactly the same results and use the same testing strategy. 5. To develop the improved solution, if it is not optimal. Once the improved solution has been obtained, the next step is to go back to 3 in summary of VAM method.

1. The Stepping-Stone Solution Method

- **The Minimum Cell Cost Solution**
  - Once an initial solution is derived, the problem must be solved using either the stepping-stone method or the modified distribution method (MODI).
  - The initial solution used as a starting point in this problem is the minimum cell cost method solution because it had the minimum total cost of the three methods used

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

Table 7:The Minimum Cell Cost Solution

- **The Allocation of One Ton from Cell 1A**

The stepping-stone method determines if there is a cell with no allocation that would reduce cost if used

From \ To	A	B	C	Supply
1	+1 6	−1 8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600
		99		

Table 8: The Allocation of One Ton from Cell 1A

- **The Subtraction of One Ton from Cell 1B**

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- Must subtract one ton from another allocation along that row.

To / From	A	B	C	Supply
1	+1 (6)	−1 (8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	(5) 75	(12)	275
Demand	200	100	300	600
99				

Table 9: The Subtraction of One Ton from Cell 1B

- **The Addition of One Ton to Cell 3B and the Subtraction of One Ton from Cell 3A**

-A requirement of this solution method is that units can only be added to and subtracted from cells that already have allocations, thus one ton must be added to a cell as shown.

To / From	A	B	C	Supply
1	+1 (6)	−1 (8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	−1 (4) 200	+1 (5) 75	(12)	275
Demand	200	100	300	600

Table 10: The Addition And Subtraction of Cells

- **The Stepping-Stone Path for Cell 2A**

-An empty cell that will reduce cost is a potential entering variable.

-To evaluate the cost reduction potential of an empty cell, a closed path connecting used cells to the empty cells is identified

To / From	A	B	C	Supply
1	(6)	− (8) 25	+ (10) 125	150
2	+ (7)	(11)	− (11) 175	175
3	− (4) 200	+ (5) 75	(12)	275
Demand	200	100	300	600
2A → 2C → 1C → 1B → 3B → 3A + $7 − 11 + 10 − 8 + 5 − 4 = −$1				

Table 11: The Stepping-Stone Path for Cell 2A

- **The Stepping-Stone Path for Cell 2B**

To / From	A	B	C	Supply
1	(6)	+ (8) 25	− (10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	− (5) 75	+ (12)	275
Demand	200	100	300	600
3C → 1C → 1B → 3B + $12 − 10 + 8 − 5 = +$5				

Table 12: The Stepping-Stone Path for Cell 2B

- **The Stepping-Stone Path for Cell 3C**

Table 13: The Stepping-Stone Path for Cell 3C

To / From	A	B	C	Supply
1	(6)	+ (8) 25	− (10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	− (5) 75	+ (12)	275
Demand	200	100	300	600
3C → 1C → 1B → 3B + $12 − 10 + 8 − 5 = +$5				

- **The Stepping-Stone Path for Cell 1A**

-After all empty cells are evaluated, the one with the greatest cost reduction potential is the entering variable.

-A tie can be broken arbitrarily

To / From	A	B	C	Supply
1	+ (6)	− (8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	− (4) 200	+ (5) 75	(12)	275
Demand	200	100	300	600

Table14: The Stepping-Stone Path for Cell 1A

- **The Second Iteration of the Stepping-Stone Method**

-When reallocating units to the entering variable (cell), the amount is the minimum amount subtracted on the stepping-stone path.

-At each iteration one variable enters and one leaves (just as in the simplex method)

To / From	A	B	C	Supply
1	(6) 25	(8)	(10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 175	(5) 100	(12)	275
Demand	200	100	300	600

Table 15: The Second Iteration of the Stepping-Stone Method

- **The Stepping-Stone Path for Cell 2B**

To / From	A	B	C	Supply
1	− (6) 25	+ (8)	(10) 125	150
2	(7)	(11)	(11) 175	175
3	+ (4) 175	− (5) 100	(12)	275
Demand	200	100	300	600
1B → 3B → 3A → 1A + $8 − 5 + 4 − 6 = +$1				

Table 16: The Stepping-Stone Path for Cell 2B

- **The Stepping-Stone Path for Cell 1B**

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To / From	A	B	C	Supply
1	6 / 25	8	10 / 125	150
2	7	11	11 / 175	175
3	4 / 175	5 / 100	12	275
Demand	200	100	300	600
1B → 3B → 3A → 1A + $8 − 5 + 4 − 6 = +$1				

Table 17: The Stepping-Stone Path for Cell 1B

- **The Stepping-Stone Path for Cell 2B**

-Continuing check for optimality

To / From	A	B	C	Supply
1	6 / 25	8	10 / 125	150
2	7	11	11 / 175	175
3	4 / 175	5 / 100	12	275
Demand	200	100	300	600
2B → 3B → 3A → 1A → 1C → 2C + $11 − 5 + 4 − 6 + 10 − 11 = +$3				

Table 18: The Stepping-Stone Path for Cell 2B

- **The Stepping-Stone Path for Cell 3C**

To / From	A	B	C	Supply
1	6 / 25	8	10 / 125	150
2	7	11	11 / 175	175
3	4 / 175	5 / 100	12	275
Demand	200	100	300	600
3C → 3A → 1A → 1C + $12 − 4 + 6 − 10 = +$4				

Table 19: The Stepping-Stone Path for Cell 3C

- The stepping-stone process is repeated until none of the empty cells will reduce
Costs (i.e., an optimal solution)
- In example, evaluation of four paths indicates no cost reductions; therefore Table 16 solution is optimal.
- Solution and total minimum cost:
x1A = 25 tons,
x2C = 175 tons,
x3A = 175 tons,
x1C = 125 tons,
x3B = 100 tons
Z = 6(25) + 8(0) + 10(125) + 7(0) + 11(0) + 11(175) + 4(175) + 5(100) + 12(0)
= 4,525

- **The Stepping-Stone Solution Method Summary**
  - i). Determine the stepping-stone paths and cost changes for each empty cell in the tableau.
  - ii). Allocate as much as possible to the empty cell with the greatest net decrease in cost.
  - iii). Repeat steps 1 and 2 until all empty cells have positive cost changes that indicate an optimal solution.

4. CONCLUSION

The transportation cost is an important element of the total cost structure for any business .The transportation problem was formulated as a Linear Programming and solved with the standard LP solvers such as the Management scientist module to obtain the optimal solution.
The computational results provided the minimal total transportation cost and the values for the decision variables for optimality. Upon solving the LP problems by the computer package, the optimum solutions provided the valuable information such as sensitivity analysis for Guinness Ghana Ltd to make optimal decisions
Through the use of this mathematical model (Transportation Model) the business (GGBL) can identify easily and efficiently plan out its transportation, so that it can not only minimize the cost of transporting goods and services but also create time utility by reaching the goods and services at the right place ad right time.
Here we have discussed about two methods by which we can minimize the total transportation costs. The methods are **Vogel Approximation Method** and the **stepping stone method**. Out of these two Methods the **stepping-stone Method** is better as it provides the least transportation cost as compared to the **Vogel Approximation**

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