International Journal of Engineering, Pure and Applied Sciences,
Vol. 2, No. 3, Sept-2017

Satellites: Gravitational field of attraction and Keplerian orbit

Sushil Chandra Karna
Department of Mathematics, M.B.M. campus, Rajbiraj, T.U. Nepal.
Email: sushilkarna22@gmail. Com

Abstract- The paper deals with the centre of mass of a system of two satellites connected by an extensible string in the gravitational field of attraction moves along a keplerian elliptical orbit.

Keywords: Satellites, Keplerian orbit , Attraction, Vectors

1. INTRODUCTION

Let the motion of a system of two satellites connected by a light, flexible and extensible string in the central gravitational field of the earth. The two satellites are assumed to be particle of masses m_1 and m_2 respectively. Suppose $\vec{r_1} and \vec{r_2}$ as their radii vectors with respect to the colure of the earth. By applying the Lagrange's equation of motion of first kind in the case of the equations of motion of the particles of masses m_1 & m_2 in the form

$$m_1\vec{r_1} + \frac{m_1\mu\vec{r_1}}{r_1^3} + \lambda\left[\frac{\left|\vec{r_1}-\vec{r_2}\right| - \ell_0}{\ell_0}\right]\frac{\left(\vec{r_1}-\vec{r_2}\right)}{\left|\vec{r_1}-\vec{r_2}\right|} = 0 \qquad (1)$$

$$m_2\vec{r_2} + \frac{m_2\mu\vec{r_2}}{r_2^3} - \lambda\left[\frac{\left|\vec{r_1}-\vec{r_2}\right| - \ell_0}{\ell_0}\right]\frac{\left(\vec{r_1}-\vec{r_2}\right)}{\left|\vec{r_1}-\vec{r_2}\right|} = 0$$

Where λ = Hook's modulus of elasticity
μ = the product of gravitational constant with the mass of the attracting churls
ℓ_0 = natural length of the string connecting the two satellites of masses m_1 & m_2. The condition of constraint is given by: $\left|\vec{r_1}-\vec{r_2}\right|^2 \le \ell_0^{\,2}$ (2)

Nature:-

1. If the inequality sign holds the $\lambda = 0$ then the motion takes place with loose string.
2. If the inequality sign holds then $\lambda \ne 0$ then the motion takes place with tight string and consequently tension in the string comes into the play.

2. MATHEMATICAL APPROACH

Suppose $\vec{R} =$ be radius vector of the colure of mass then

$$\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2} \qquad (3)$$

From equation (i) by adding

$$M\vec{R} + \mu\left(\frac{m_1\vec{r_1}}{r_1^3} + \frac{m_2\vec{r_2}}{r_2^3}\right) \qquad (4)$$

Where

$M = m_1 + m_2$

Our assumption that the maximum extended length ℓ_E of the string is infinitesimally small compared to the distances r_1 and r_2 of the particles from the colure of force

$$\left.\begin{aligned} \frac{\ell_E}{r_1} &<< \in \\ \frac{\ell_E}{r_2} &<< \in \end{aligned}\right\} \qquad (5)$$

Suppose $\vec{P_1} and \vec{P_2}$ be the radius vectors of the particles of masses m_1 and m_2 respectively with origin at the culver of mass

$$\left.\begin{aligned} \vec{r_1} &= \vec{R} + \vec{P_1} \\ \vec{r_2} &= \vec{R} + \vec{P_2} \end{aligned}\right\} \qquad (6)$$

Obviously,

$$p_1 < \ell_E \qquad p_2 < \ell_E$$

$$:- \quad p_1 << r_1 \ \& \ p_2 << r_2$$

$$r_1 \approx r_2 \approx R$$

$$:- \quad \frac{p_1}{R} << \in \& \frac{p_2}{R} << \in$$

International Journal of Engineering, Pure and Applied Sciences,
Vol. 2, No. 3, Sept-2017

Eliminating $\vec{r_1} and \vec{r_2}$ from (4) with the help of (6) and expanding in ascending power of small quantities $\frac{p_1}{R}$ & $\frac{p_2}{R}$ we get

$$M\vec{R} + \mu\frac{M\vec{R}}{R^3} = \vec{F_1} + \vec{F_2} + O_3 \quad (8)$$

Where

$$\vec{F_1} = \frac{3\mu}{R^3}\left(m_1\vec{p_1} + m_2\vec{p_2}\right) - \frac{3\mu}{Rs}\left[\vec{R}.(m_1\vec{p_1} + m_2\vec{p_2})\right]\vec{R}$$

$$\vec{F} = \frac{-3\mu}{2R^3}\left[m_1\left\{\left(\frac{\vec{p_1}}{R}\right)^2 - 5\left(\frac{\vec{R}}{R}.\frac{\vec{p}}{R}\right)^2\right\} + m_2\left\{\left(\frac{\vec{p_2}}{R}\right)^2 - 5\left(\frac{\vec{R}}{R}.\frac{\vec{p_2}}{R}\right)^2\right\}\right].R - \frac{3\mu m_1}{R^5}\left(\vec{R}.\vec{P_1}\right)\vec{P_1} + \frac{3\mu}{R^5}m_2\left(\vec{R}.\vec{P_2}\right)\vec{p_2} \quad (9)$$

0_3= third and higher order terms be neglected
$\in$ = arbitrary positive number ($\in$ = 1 say)

We obtain by the equation (3) & (6)

$$\vec{p_1} = \frac{m_2}{m_1 + m_2}(\vec{r_1} - \vec{r_2}) \quad (10)$$

$$\vec{p_2} = \frac{m_1}{m_1 + m_2}(\vec{r_2} - \vec{r_1}) \quad (11)$$

from (10) & (11)

$$m_1\vec{p_2} + m_2\vec{p_2} = 0 \quad (12)$$

Therefore $\vec{F_1}$ given (9) vanishes and neglecting the second and higher order perturbation terms in (9)
The equation of colure of mass given by (8) takes the form

$$\mathrm{M}\vec{R} + \frac{\mu M\vec{R_1}}{R^3} = 0 \text{ .... } (13)$$

The equation (13) shows that the colure of mass of the system can be assumed to move along a Keplerian elliptical orbit with the higher degree of accuracy up to 2nd order infinitesimal in $\frac{P_1}{R} and \frac{P_2}{R}$

CONCLUSION

The centre of mass of a system of two Satellites connected by an extensible string in the central gravitational field of attraction moves along a given keplerian elliptical orbit

REFERENCES

[1] Bate, R.R Mueller, D.D and white, J.E fundamentals of Astrodynamies Dover, New York 1671
[2] Danby, J.M.A Fundamentals of celestial Mechanics Mevemillan, New York, 1962
[3] Battin, R.H. Astronautical Guidance Mcgraw-Hill, New York, 1964
[4] Fitzpatrek, P.M. Principles of celestial Mechanics Dcademic press, New York, 1970
[5] Kaplan, M.H. Modern spaceeraft Dynamics and coulrol, Wiley, New York, 1976
[6] Meirovitch, L, Mcthods of Analyfical Dynauics Mcgraw-Hill, New York, 1970.
[7] Hughes, Peter, c. Spacecraft Altitude Dynamics wile, New York, 1986.
[8] Kane yeoman R, Likens, Peter L, and Evensong, D.A. Spacecraft Dynamics MC-Grew Hill, New-York, 1983.
[9] Thomson, W.T. Involution to space Dynamics hilly, New York 1961.
[10] Draper C.S, wrangle, W, and Hookah, H inertial Guidance, paragon, New York 1960
[11] Bruiting, K.R Inertial Navigation System Dialysis Wiley, New York. 1971.
[12] Meirovitch L. Methods of Analytical Dynamics MC-Grew –Hill, New York
[13] Chakrabarti, S. (2000): Data mining for hypertext: A tutorial survey. SIGKDD explorations, **1**(2), pp. 1–11.