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Scheduled various Maps regarding Beta-Sets and Allied Groups

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Abstract- The concept of group of functions, say $\beta ch(X,\tau)$ preserving β-closed sets containing homeomorphism group $h(X,\tau)$ was studied by Arora, Tahiliani and Maki. In continuation to that, we study some new isomorphisms, mappings, subgroups and their properties.

Keywords: α-open, β-open and β-irresolute mappings.

1. INTRODUCTION AND PRELIMINARIES

Throughout this paper we consider spaces on which no separation axiom are assumed unless explicitly stated.The topology of a space(By space we always mean a topological space) is denoted by τ and (X,τ) will be replaced by X if there is no chance of confusion. For $A\subseteq X$, the closure and interior of A in X are denoted by Cl(A) and Int(A) respectively. Let A be a subset of the space (X,τ).Then A is said to be β-open [1] if $A\subseteq Cl(Int(Cl(A)))$. Its complement is β-closed. The family of all β- open sets containing A is denoted by βO(A) and all β-closed sets containing A is denoted by βC(A). A is said to be α-open[6] if $A\subseteq Int(Cl(Int(A)))$ and its complement is α-closed. The union of all β-open sets contained in A is called β- interior of A, denoted byβInt (A)[2].

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called β-irresolute [4] if the inverse image of every β-open set in Y is β-open in X. It is called βc-homeomorphism [5] if f is β-irresolute bisection and f^{-1} isβ-irresolute.

2. SUBGROUPS OF BCH(X; T)

For a topological space (X,τ) we have $h(X;\tau) = \{f \mid f:(X,\tau)\rightarrow(X,\tau)$ is ahomeomorphism$\}$[5] and $\beta ch(X;\tau)=\{f \mid f:(X,\tau)\rightarrow(X,\tau)$ is a βc homeomorphism$\}$[5].

In this section, we investigate some structures of $\beta ch(H;\tau|H)$ for a subspace $(H,\tau|H)$ of (X,τ) using two subgroups of $\beta ch(X,\tau)$, say $\beta ch(X, X \setminus H; \tau)$ and $\beta ch_0(X, X \setminus H; \tau)$ below.

Definition 2.1. For a topological space (X,τ) and subset H of X, we define the following families of maps:

(i). $\beta ch(X, X \setminus H; \tau)=\{a \mid a\in \beta ch(X;\tau)$ and $a(X \setminus H)= X \setminus H\}$.

(ii). $\beta ch_0(X, X \setminus H; \tau)= \{a \mid a\in \beta ch(X, X \setminus H; \tau)$ and $a(x)=x$ for every $x\in X \setminus H\}$.

Theorem 2.2. Let H be a subset of a topological space (X,τ).Then

(i) The family $\beta ch(X, X \setminus H; \tau)$ forms a subgroup of$\beta ch(X,\tau)$.

(ii) The family $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X, X \setminus H; \tau)$ and hence $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of$\beta ch(X,\tau)$.

Proof. (i). It is shown obviously that $\beta ch(X, X \setminus H; \tau)$ is a non-empty subset of $\beta ch(X,\tau)$, because $1_X\in \beta ch(X, X \setminus H; \tau)$. Moreover, we have that $\omega_X(a,b^{-1})=b^{-1}o\ a\in \beta ch(X, X \setminus H; \tau)$ for any elements $a, b\in \beta ch(X, X \setminus H; \tau)$,where $\omega_X = \omega|(\beta ch(X, X \setminus H; \tau) \times \beta ch(X, X \setminus H; \tau))$ as ω is the binary operation of the group $\beta ch(X,\tau)$. Evidently, the identity map 1_X is the identity element of $\beta ch(X, X \setminus H;\tau)$.

(ii). It is shown that $\beta ch_0(X, X \setminus H; \tau)$ is a non-empty subset of $\beta ch(X, X \setminus H; \tau)$ because $1_X\in \beta ch_0(X, X \setminus H; \tau)$.We have that $\omega_{X,0}(a,b^{-1})=b^{-1}o\ a\in \beta ch_0(X, X \setminus H; \tau)$ for any elements $a, b\in \beta ch_0(X, X \setminus H; \tau)$,where $\omega_{X,0}=\omega_X|(\ |(\beta ch_0(X, X \setminus H; \tau) \times \beta ch_0(X, X\setminus H; \tau))(\ \omega_X$ is the binary

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operation of the group βch(X, X \ H; τ)).Thus βch0(X, X \ H; τ) is a subgroup of βch(X, X \ H; τ) and the identity map 1X is the identity element of βch0(X, X \ H; τ). By using (i), βch0(X, X \ H; τ) forms a subgroup of βch(X,τ). Let H and K be the subsets of X and Y respectively. For a map f:X→Y satisfying a property K=f(H) ,we define the following map rH,K(f):H→K by rH,K(f)(x)=f(x) for every x∈H. Then, we have that jKo rH,K(f)= f|H:H→Y,where jK:K→Y be an inclusion defined by jK(y)=y for every y∈K and f|H:H→Y is a restriction of f to H defined by (f|H)(x)=f(x) for every x∈H. Especially, we consider the following case that X=Y, H=K⊆X and a(H)=H ,b(H)=H for any maps a,b:X→X.Thus rH,H(boa)= rH,H(b) o rH,H(a) holds. Moreover ,if a map a:X→X is a bisection such that a(H)=H, then rH,H:H→H is bijective and rH,H(a^{-1})=(rH,H(a))$^{-1}$.

We recall well known properties on β-open sets of subspace topological spaces.

Theorem 2.3. For a topological space (X,τ) and subsets H and U of X and A⊆H,V⊆H and B⊆H, the following properties hold:

(i). Arbitrary union of β-open sets of (X,τ) is β-open in (X,τ).The intersection of an open set of (X,τ) and a β-open set in (X,τ) is β-open in (X,τ).

(ii). (a). If A is β-open in (X,τ) and A⊆H, then A is β-open in a subspace (H,τ|H).

(b). If H⊆X is open or α-open in (X,τ) and a subset U⊆X is β-open in (X,τ), then H∩U is β-open in asubspace (H,τ|H).

(iii). Let V⊆ H⊆X.

(a). If H is β-open in (X,τ),then IntH(V) ⊆ βInt(V) holds.

(b). If H is β-open in (X,τ) and V is β-open in a subspace (H,τ• H) then V is β-open in (X,τ).

Let B• H• X. If H is β-closed in (X,τ) and B is β-closed in a subspace (H,τ• H), then B is β-closed in(X,τ).

(a). Assume that H is a open subset of (X,τ).Then,βO(X,τ)• H• βO(H,τ• H) holds, where βO(X,τ)• H={W• H• W• βO(X,τ)}.

(b). Assume that H is a β-open subset of (X,τ) .Then,βO(H,τ• H) • βO(X,τ)• Hholds.

(c). Assume that H is a β-open subset of (X,τ).Then, βO(H,τ• H) = βO(X,τ)• Hholds.

Proof. (i).Clear from Remark 1.1 of [1] and Theorem 2.7 of[3].

(ii).(a). Clear.(ii-2).Its Lemma 2.5 of [1].

(iii)(a). Let x• IntH(V).There exists a subset W(x) ∈ τ such that W(x) ∩H⊆V. By (i), W(x) ∩H∈ βO(X,τ).This shows that x∈ βInt(V) and so IntH(V) ⊆ βInt(V).

(b) and (iv).Its clear from Lemma 2.7 of [1].

(v). (b). Let V∈ βO(X,τ)|H. For some set W∈ βO(X,τ),V=W∩H and so we have W∩H∈ βO(H,τ|H)(from ii-2). Hence V∈βO(H,τ|H) holds.

(b). Let V∈βO(H,τ|H).Since H∈ βO(X,τ),we have V∈ βO(X,τ) by (iii-2).Thus V= V∩H∈ βO(X,τ)|H.

(c). It follows from (v-1) and (v-2).

Lemma 2.4. (i).If f:(X,τ) →(Y,σ) is β-irresolute and a subset H is α-open in (X,τ),then f|H:(H,τ|H) →(Y,σ) isβ-irresolute.

Let (1) and (2) be properties of two maps k:(X,τ) →(K,σ|K),where K⊆Y, and jKok: (X,τ) →(Y,σ) asfollows:

1) k:(X,τ) →(K,σ|K) isβ-irresolute.
2) jKok: (X,τ) →(Y,σ) isβ-irresolute.

Then, the following implication and equivalence hold:

(a). Under the assumption that K is α-open in (Y,σ),(1)⇒(2).

(b). Conversely, under the assumption that K is β-open in (Y,σ) ,(2)⇒(1). (ii-3). Under the assumption that K is β-open in (Y,σ),(1)⇔(2).

(ii). If f:(X,τ) →(Y,σ) is β-irresolute and a subset H is α-open in (X,τ) and f(H) is β-open in (Y,σ),then r H,f(H)(f):

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(H,τ|H) →(f(H), σ|f(H)) isβ-irresolute.

Proof.(i). Let V∈βO(Y,σ).Then,we have $(f\mid H)^{-1}(V)=f^{-1}(V)\cap H$ and $(f\mid H)^{-1}(V)\in$ βO(H,τ|H).(Theorem 2.3 (ii-2)).

(ii).(a) (1)⇒(2).Let V∈ βO(Y,σ).Since $(jKok)^{-1}(V)=k^{-1}(V\cap K)$ andV∩K∈ βO(K,σ|K) (Theorem 2.3 (ii-2)),we have that $(jKok)^{-1}(V)\in$ βO(X,τ) and hence jKok is β-irresolute.

(b) (2) ⇒(1).Let U∈ βO(K,σ|K).Since U∈βO(Y,σ)(Theorem 2.3 (iii-2)),we have $k^{-1}(U)=(jKok)^{-1}(U)\in$βO(X,τ).Thus k is β-irresolute.

(c). Obvious in the view of fact that every α-open set is β-open,it is obtained by (ii-1) and(ii-2).

(iii). By (i), f|H:(H,τ|H) →(Y,σ) is β-irresolute.The map r H,f(H)(f) is β-irresolute, because f|H=jf(H)o r H,f(H)(f)holds.

Definition 2.5. For an α-open subset H of (X,τ), the following maps(rH)*: βch(X, X \ H; τ) → βch(H;τ|H) and (rH)*,0: βch0(X, X \ H; τ) →βch(H;τ|H) are well defined as follows (Lemma 2.4 (iii)),respectively:

(rH)*(f)= rH,H(f) for every f∈ βch(X, X \ H; τ);

(rH)*,0(g)= rH,H(g) for every g∈ βch0(X, X \ H; τ).Indeed ,in Lemma 2.4 (iii),we assume that X=Y, τ=σ and H=f(H).

Then, under the assumption that H is α-open hence β-open in (X,τ), it is obtained that rH,H(f) ∈ βch(H;τ|H) holds for any f∈βch(X, X \ H; τ)(resp. f∈ βch0(X, X \ H; τ)).

We need the following lemma and then we prove that (rH)* and (rH)*,0are onto homomorphisms under the assumptions that H is α-open and α-closed in (X,τ).

Let X=U1∪U2 for some subsets U1 and U2 and f1:(U1,τ|U1) →(Y,σ)and

f2:(U2,τ|U2) →(Y,σ) be the two maps satisfying a property f1(x)=f2(x) for every x∈ U1∩U2.Then, a map f1∇f2 is well defined as follows:
(f1∇f2)(x)= f1(x) for every x∈ U1 and (f1∇f2)(x)= f2(x) for every x∈ U2. We call this map a combination of f1 and f2.

Lemma 2.6. For a topological space (X,τ),we assume that X= U1∪U2,where U1 and U2 are subsets of X and f1:(U1,τ|U1) →(Y,σ) and f2:(U2,τ|U2) →(Y,σ) be the two maps satisfying a property f1(x)=f2(x) for every x∈ U1∩ U2.Then if Ui∈βO(X,τ) for each i∈{1,2} and f1 and f2 are β-irresolute, then its combination f1∇f2 : (X,τ) →(Y,σ) is β- irresolute.

Proof. Its on similar lines in ([1], Theorem 2.8).

Theorem 2.7. Let H be a subset of a topological space (X,τ).
(i).(a). If H is α-open in (X,τ),then the maps (rH)*: βch(X, X\H;τ) → βch(H;τ|H) and (rH)*,0: βch0(X, X \ H; τ) → βch(H;τ|H) are homomorphism of groups.Morever (rH)* | βch0(X, X \ H; τ)= (rH)*,0 holds(Definition 2.5).
(b). If H is α-open and α-closed in (X,τ),then the maps (rH)*: βch(X, X\H; τ) → βch(H;τ|H) and (rH)*0: βch0(X, X \ H; τ) → βch(H;τ|H) are onto homomorphism of groups.
(ii).For an α-open subset H of (X,τ),we have the following isomorphisms of groups: (ii-1). βch(X, X \ H; τ) |Ker(rH)* is isomorphic to Im(rH)*;
(b). βch0(X, X \ H; τ) is isomorphic to Im(rH)*,0 holds.where Ker(rH)*={a∈ βch(X, X \ H; τ) |(rH)*(a)=1X} is a normal subgroup of βch(X, X \ H; τ); Im(rH)*={(rH)*(a) | a∈ βch(X, X \ H; τ)}and Im(rH)*,0={(rH)*,0 (b) | b∈ βch0(X, X \ H; τ)}are subgroups of βch(X,τ).
(iii). For an α-open and α-closed subset H of (X,τ),we have the following isomorphisms of groups:
(c).βch(H;τ|H) is isomorphic to βch(X, X \ H; τ) |Ker(rH)*. (iii-2). βch(H;τ|H) is isomorphic to βch0(X, X \ H; τ).

Proof. (i).
(a). Let a,b∈ βch(X, X \ H; τ). Since H is α-open in (X,τ),the maps (rH)* and (rH)*,0 are well defined(Definition 2.5).Then we have that (rH)*(ωX(a,b))= (rH)*(boa)= rH,H(boa)= rH,H(b) o rH,H(a)= ωX((rH)*(a), (rH)*(b)) hold, where ωH is a binary operation of βch(H;τ|H) ([5] Theorem 4.4 (iv)).Thus (rH)* is a homomorphism of groups. For the map (rH)*0: βch0(X, X \ H; τ) → βch(H;τ|H),we

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have that $(r_H)^*{}_{,0}(\omega_{X,0}(a,b))= (r_H)^*{}_{,0}(b\circ a)= r_{H,H}(b\circ a)= r_{H,H}(b)\circ r_{H,H}(a)= \omega_X((r_H)^*(a), (r_H)^*(b))$ hold, where ω_X is a binary operation of $\beta ch(H;\tau|H)$(Theorem 2.3 (ii)).Thus $(r_H)^*{}_{,0}$ is also a homomorphism of groups. It is obviously shown that $(r_H)^* \mid \beta ch_0(X, X \setminus H; \tau)= (r_H)^*{}_{,0}$ holds.(Definitions 2.1 and 2.5).

(b).In order to prove that $(r_H)^*$ and $(r_H)^*{}_0$ are onto , let $h\in \beta ch(H;\tau|H)$. Let j_H: $(H;\tau|H) \to(X,\tau)$ and $J_{X \setminus H}$: $(X \setminus H, \tau| X \setminus H) \to(X,\tau)$ be the inclusions defined $j_H(x)=x$ for every $x\in H$ and $J_{X \setminus H}(x)=x$ for every $x\in X \setminus H$. We consider the combination $h_1=(j_H o h)\ \nabla(j_{X \setminus H} o 1_{X \setminus H})$: $(X,\tau) \to(X,\tau)$. By Lemma 2.4 (ii-1),under the assumption of α-openness on H, it is shown that two maps $j_H\, oh$: $(H;\tau|H) \to(X,\tau)$ and $j_H\, oh^{-1}$: $(H;\tau|H) \to(X,\tau)$ are β-irresolute; moreover under the assumption of α-openness on$X\setminus H$, $J_{X \setminus H}\ o\ 1_{X \setminus H}$: $(X \setminus H,\tau| X \setminus H) \to(X,\tau)$ is β-irresolute. Using lemma 2.6, for a β- open cover $\{H, X \setminus H\}$ of X, the combination above h_1: $(X,\tau) \to(X,\tau)$ is β-irresolute. Since h_1 is bijective, its inverse map $h_1{}^{-1}=(j_H oh^{-1})\ \nabla(j_{X \setminus H}\ o\ 1_{X \setminus H})$ is also β- irresolute. Thus under the assumption that both H and $X \setminus H$ are β-open in (X,τ), we have $h_1\in \beta ch(X,\tau)$. Since $h_1(x)=x$ for every point $x\in X \setminus H$, we conclude that $h_1\in\beta ch_0(X, X \setminus H; \tau)$ and so $h_1\in\beta ch(X, X \setminus H; \tau)$. Moreover, $(r_H)^*{}_{,0}(h_1)= (r_H)^*(h_1)= r_{H,H}((h_1)=h$, hence $(r_H)^*$ and $(r_H)^*{}_{,0}$ are onto, under the assumption that H is α-open and α-closed subset of(X,τ).

(ii).By (a) above and the first isomorphism theorem of group theory, it is shown that there are group isomorphism below, under the assumption that H is α-open in (X,τ):(*). $\beta ch(X, X \setminus H; \tau)\ |Ker(r_H)^*$ is isomorphic to $Im(r_H)^*$; and (**). $\beta ch_0(X, X \setminus H; \tau)|Ker(r_H)^*{}_0$ is isomorphic to $Im(r_H)^*{}_{,0}$where $Ker(r_H)^*{}_0 =\{a\in\beta ch_0(X, X \setminus H; \tau)\ |(r_H)^*{}_0(a)=1_X\}$. Moreover, under the assumption of α-openness on H, it is shown that $Ker(r_H)^*{}_0 =\{1_H\}$.Therefore, using (**) above, we have the isomorphism(ii-2).

(iii).By (b) above, it is shown that $(r_H)^*$ and $(r_H)^*{}_{,0}$ are onto homomorphism of groups, under the assumption that H is α-open and α-closed in (X,τ).Therefore ,by (ii) above, theisomorphisms (iii-1) and (c) are obtained.

Remark 2.8. Under the assumption that H is α-open and α-closed in (X, τ), Theorem2.7(iii)isproved.Let(X,τ)beatopologicalspacewhere$X=\{a,b,c\}$and$\tau=\{\varphi,X, \{a\},\{b,c\}\}$, and $(H;\tau|H)$ is a subspace of (X,τ),where $H=\{a\}$.Then $\beta O(X,\tau)=P(X)$(the power set of X) and H is α -open and α -closed in (X,τ).We apply Theorem 2.7 (iii) to the present case, we have the group isomorphisms. Directly, we obtain the following date on groups: $\beta ch(X,\tau)$ is isomorphic to S_3,the symmetric group of degree 3, $\beta ch(X, X\setminus H;\tau)=\{1_X,h_a\}$,$Ker(r_H)^*=\{1_X,h_a\}$,$\beta ch(H;\tau\setminus H) =\{1_H\}$andso$\beta ch_0(X,X\setminus H;\tau)= \{1_X\}$,where $h_a:(X,\tau) \to(X,\tau)$ is a map defined by $h_a(a)=a$, $h_a(b)=c$ and $h_a(c)=b$. Therefore in this example, we have $\beta ch(H;\tau|H)$ is isomorphic to $\beta ch(X, X \setminus H;\tau) |Ker(r_H)^*$ and $\beta ch(H;\tau|H)$ is isomorphic to $\beta ch_0(X, X \setminus H; \tau)$. Moreover we have $h(X,\tau)=\{1_X, h_a\}$.

(iii). Even if a subset H of a topological space (X,τ) is not α-closed and it is α-open, we have the possibilities to investigate isomorphisms of groups corresponding to a subspace $(H, \tau|H)$ and $(r_H)^*$ using Theorem 5.7(ii). For example, Let (X,τ) be a topological space where $X=\{a,b,c\}$ and $\tau=\{\varphi, X, \{a,b\}\}$,and $(H;\tau|H)$ is a subspace of (X,τ),where $H=\{a,b\}$.Then $\beta O(X,\tau)=P(X)$(the power set of X) and H is α-open butnot α-closed in (X,τ). By theorem 2.7(i)(i-1), the maps $(r_H)^*$: $\beta ch(X, X \setminus H;\tau) \to \beta ch(H;\tau|H)$ and $(r_H)^*{}_{,0}$: $\beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$ are homomorphism of groups and by theorem 5.7(ii) two isomorphisms of groups are obtained:

(*-1). $\beta ch(X, X \setminus H; \tau)/Ker(r_H)^*$ is isomorphic to $Im(r_H)^*$. (*-2). $\beta ch_0(X, X \setminus H; \tau)/Ker(r_H)^*$ is isomorphic to $Im(r_H)^*{}_{,0}$.

We need notation on maps as follows: let h_c: $(X,\tau) \to(X,\tau)$ and $t_{a,b}$: $(H,\tau|H) \to (H,\tau|H)$ are

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the maps defined by h_c (a)=b, h_c(a)=b, h_c(c)=c and $t_{a,b}$(a)=b, $t_{a,b}$(b)=a, respectively.Then it is directly shown that βch(X, X \ H; τ)={1_X, h_c}which is isomorphic to Z_2, $(h_c)^2$= 1_X, and Ker$(r_H)^*$={a∈βch(X, X \ H; τ)| $(r_H)^*$(a)= 1_H}={a∈{1_X, h_c}|$(r_H)^*$(a)= 1_H }={1_X }because $(r_H)^*$(1_X)= 1_H and $(r_H)^*$(h_c)= $t_{a,b}$not equalto1_H .Byusing(*-1)above,Im$(r_H)^*$isisomorphictoβch(X,X\H;τ)={ 1_X,h_c}and so Im$(r_H)^*$={ 1_H, $r_{H,H}(h_c)$}={1_H, $t_{a,b}$}.Since Im$(r_H)^*$⊆ βch(H;τ|H) ⊆{1_H, $t_{a,b}$},we have that Im$(r_H)^*$ = βch(H,τ|H)= {1_H, $t_{a,b}$} and hence $(r_H)^*$is onto. Namely, we have an isomorphism $(r_H)^*$: βch(X, X \ H; τ) is isomorphic to βch(H ;τ|H) which is isomorphic to Z_2.Moreover it is shown that $βch_0$(X, X \ H; τ)={a∈βch(X, X \ H; τ)|a(x)=x for any x∈{c}}={1_X, h_c}= βch(X, X \ H; τ) hold and so $(r_H)^*$= $(r_H)^{*,0}$holds.

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