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Cube Roots in Vedic Mathematics

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Abstract– Finding the cube roots of a number by conventional method is time consuming and tedious job with more complexity. However, using Vedic methods it becomes interesting and fast too. There are special and general methods to find cube roots in Vedic Mathematics. This article only focuses on special cases and explains how Vedic method can be applied to extract the cube roots of any perfect cube number as well as bigger and bigger numbers to make it easier for students to understand. This article only concentrates to illustrate real part of cube roots not for imaginary roots. The method for determining the cube root was firstly provided by the great Indian Mathematician or astronomer Aryabhatta in 556 B.S. and father of Vedic Mathematics Bharati Krishna Tirthaji Maharaja's method is likely to Symmetry with Aryabhatta's method. Here this article focuses on only the Vedic methods not on Aryabhatta's.

Key Words:– Cube Roots; Perfect cube; विलोकनम; विजाङ्क; नवशेष; Vedic Mathematics; Aryabhatta.

1. INTRODUCTION

The third power of the number is called its cube, and the inverse operation of finding a number whose cube is n is called extracting the cube root of n. In mathematics, a cube root of a number x is a number y such that $y^3 = x$. In Mathematics, each real number (except 0) has exactly one real cube root (which is known as principal cube root) and two imaginary cube roots, and all non-zero complex number have three distinct complex (imaginary) cube roots. For example, the cube root of 8 are: 2, $-1 + i\sqrt{3}$, $-1 - i\sqrt{3}$ whereas cube roots of 8i are: -2i, $\sqrt{3} + i$, $-\sqrt{3} + i$. Thus, the cube root of a number x are the numbers of which satisfy the equation $y^3 = x$.

The present existing curriculum taught in our school level uses only the method of factorization of a number to extract cube roots. Finding the cube roots of a number by conventional method is time consuming and tedious job with more complexity. However, using Vedic methods it becomes interesting and fast too. We can find the cube root of any number within very short period by just looking at the number with perfect practice. There are special and general methods for calculating cube root of the number. There is a general formula for n terms, where n being any positive numbers. Special parts also specific for special numbers of cube with certain digits of numbers. This article only concentrates to illustrate the specific methods of Vedic Mathematics for real part of cube roots not for imaginary roots. It explains how Vedic method can be applied to extract the cube roots of any perfect cube number as well as bigger and bigger numbers to make it easier for students to understand. Among the 16 Sutras and 13 sub-sutras of Vedic mathematics विलोकनम and विजाङ्क are mostly used for it.

2. HISTORY OF CUBE ROOTS[1]

The method for determining the cube root was firstly provided by the great Mathematician or Astronomer Aryabhatta in 556 B.S. . Brahmagupta (in 685 B.S.), Shreedhar (in nearly 9th Century of B.S.), Shreepati (in 1096 B.S.), Bhaskar (in 1207 B.S.), Narayan (in 1413 B.S.) and other scholars followed the method of Aryabhatta's to find cube root of numbers. Aryabhatta has also mentioned in his book, गणितपाद, a method to extract the cube root of any number, but the method is too complex to understand the fifth sloka of Aryabhatta's book "गणितपाद" mentioned as:

अघनाद भजेद द्वितीयात त्रिगुणेन घनस्य मूलवर्गेण ।
वर्ग स्त्रिपूर्व गुणित : शोध्य : प्रथमाद घनस्य घनात् ॥ [2]
(आर्यभटीयको गणितपाद)

With a slight modification of method of Aryabhatta, Samanta Chandra Sekhar came up with his own way to determine the cube root in 1926 B.S. Even the modern mathematical practices follows the same way to find the cube root as Aryabhatta used, with few tweaks and turns. Bapudev Shastri was a latter scholar to Chandra Sekhar who came up with his method, accompanied by about half of the method provided by Aryabhatta in 1940 B.S.. However, 1952 B.S. became the most wonderful year for "cube root" as Gopal

[1] Pant, Nayaraj [2037 B.S.]: "पण्डित गोपाल पाण्डे र उनको घनमूल ल्याउने रीति", नेपाल राजकीय प्रज्ञा प्रतिष्ठान, pp. 86-87.
[2] Pant, Nayaraj [2037 B.S.]: "पण्डित गोपाल पाण्डे र उनको घनमूल ल्याउने रीति", नेपाल राजकीय प्रज्ञा प्रतिष्ठान, pp. 64

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Vol. 3, No. 1, March-2018

Pandey came up with the must excellent and practicable method for determining cube roots. Brahmangupta express his method for finding cube roots as:

छेदोडघनाद् द्वीतियाद् घनमूलकृतिस्त्रिसङ गुणाप्तकृतिः ।
शोध्या त्रिपूर्वगुणिता प्रथमाद् घनतो घनो मूलम् ॥ ७ ॥ [3]
(ब्राह्मस्फुटसिद्धान्त गणिताध्याय)

All the mathematician after Aryabhatta followed his method with some modification. Specially, father of Vedic Mathematics Bharati Krishna Tirthaji Maharaja's method is likely to Symmetry with Aryabhatta's method.

3. VEDIC METHODS TO FIND CUBE ROOTS OF PERFECT CUBES

To calculate cube root of any perfect cube quickly, we need to remember the cubes from1 to 10.
$1^3 = 1$; $2^3 = 8$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$
$6^3 = 216$; $7^3 = 343$; $8^3 = 512$; $9^3 = 729$; $10^3 = 1000$
From the above cubes of 1 to 10, we need to remember the following facts:

i) If the last digit of perfect cubes are 1, 4, 5, 6, 9 or 10 then the last digit of cube roots are same.
ii) If the last digit of perfect cubes are 2, 3, 7 or 8 then the last digit of cube roots are 8, 7, 3 or 2 respectively.

It's very easy to remember the relations given above as follows:

Table 1

$1 \Rightarrow 1$	Same numbers
$8 \Rightarrow 2$	Complement of 8 is 2
$7 \Rightarrow 3$	Complement of 7 is 3
$4 \Rightarrow 4$	Same numbers
$5 \Rightarrow 5$	Same numbers
$6 \Rightarrow 6$	Same number
$3 \Rightarrow 7$	Complement of 3 is 7
$2 \Rightarrow 8$	Complement of 2 is 8
$9 \Rightarrow 9$	Same number
$0 \Rightarrow 0$	Same number

i.e. $8 \Leftrightarrow 2$ and $7 \Leftrightarrow 3$, remaining are same. Moreover, it can be concluded that the cubes of the first nine natural numbers have their own distinct endings; and there is no possibility of overlapping or doubt as in the case of square.
There is another question arises that the given number is perfect cube or not? Is there any rule to notify it? Yes, the Vedic method नवशेष determined the number whose cube root is to be extracted is a perfect cube or not. Observe the table:

[3] Pant, Nayaraj [2037 B.S.]: "पण्डित गोपाल पाण्डे र उनको घनमूल ल्याउने रीति", नेपाल राजकीय प्रज्ञा प्रतिष्ठान, pp. 64

Table 2

Number	Cube	विजाङ्क of Cube (Using नवशेष)
1	1	1
2	8	8
3	27	0
4	64	1
5	125	8
6	216	0
7	343	1
8	512	8
9	729	0

From the table, it can be concluded that, if the digit sum of the number is found to be 0, 1 or 8 then the given number is a perfect cube. It should also be noted that this is necessary but not sufficient condition to test the perfect cube number. i.e. if the विजाङ्क of the cube does not match with the विजाङ्क (digit sum) of cubes of cube roots then the answer is definitely wrong. But if they match each other than the answer is most likely correct and not definitely correct.

Example: 2744 may be a perfect cube because विजाङ्क of this number is 2 + 7 + 4 + 4 = 8 (by using नवशेष method)

The left digit of a cube root having more than 7 digits, or ten's digit of cube root having less than 7 digits can be extracted with the help of the following table:

Table 3

Left-most pair of the Cube Roots	Nearest Cube Roots
1 – 7	1
8 – 26	2
27 – 63	3
64 – 124	4
125 – 215	5
216 – 342	6
343 – 511	7
512 – 728	8
729 – 999	9

4. DIFFERENT CASES FOR PERFECT CUBES

There are three cases:

(i) Cube root of a number having less than 7 digits.
(ii) Cube root of a number having more than 7 digits but less than 10 digits.
(iii) Cube root of a number greater than 7 digits but ending with even numbers.

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4.1. Rules for all three cases

I. Make group of 3 digits, starting from the right.
   (a) The numbers having 4, 5 to 6 digits will have a 2 digits cube root.
   (b) The numbers having 7, 8 or 9 digits will have a 3 digits cube root.
   (c) The number having 10, 11 or 12 digits will have a 4 digits cube root.

II. Table [1] will help us to determine the unit digit of cube root and table [3] will give the left digit of the cube root.

4.1.1. Illustration for Case I

Vilokanam (विलोकनम) method is used to extract the cube root of a number having less than 7 digits.

Example: Find the cube root of 17576

Stepwise: (a) Placing bar as: $\overline{17}\ \overline{576}$

(b) It will have a two digit cube root.

(c) The first bar falls on unit digit 6, so from above table [1] & [2], we can say that the unit digit of the root is 6.

(d) From table [3], left digit of cube root is 2.

Hence, $\sqrt[3]{17576} = 26$.

4.1.2. Illustration for Case II

The cube root of more than 7 digits number will contain 3 digits. Let it be denoted by unit digit (R), left digit (L) and middle digit (M). L and R can be determined by विलोकनम method, whereas M can be determined by विजाङ्क .

Steps: (a) Subtract R^3 from the number and eliminate the last zero.

(b) The middle digit of cube root is obtained by $3R^2M$. Substituting different values of M. So that we may reach to the unit digit of the number obtained in the previous step.

If we obtained more values of M, we use विजाङ्क method which should equal to the given number and choosing best suited.

Example: Find $\sqrt[3]{12977875}$.

Here, विलोकनम and विजाङ्क methods are used:

(a) Placing bar as: $\overline{12}\ \overline{977}\ \overline{875}$

(b) Unit digit of cube root is 5 i.e. R = 5.

(c) Here, left group is 12, so from the table [3], L = 2.

(d) Subtracting R^3 from 12977875 i.e. 12977875 – 125 and eliminate last zero is 1297775.

(e) Middle digit of cube root is obtained by $3R^2M = 3 \times 5^2 \times M = 75$ M.

(f) We should be looking for a suitable value of M so that the unit digit of 75M becomes equal to the unit digit of 1297775 (which is obtained in (d)).

If we obtain more than one value of M, we use विजाङ्क method and test which value of M is best suited in this case. In this problem, here is 5 options for M, i.e. 1, 3, 5, 7 or 9.

Where, विजाङ्क of given number 12977875 is 1.

Again, विजाङ्क of $(215)^3 = 8$; विजाङ्क of $(235)^3 = 1$; विजाङ्क of $(255)^3 = 0$;

विजाङ्क of $(275)^3 = 8$ and विजाङ्क of $(295)^3 = 1$

Here, विजाङ्क of $(235)^3$ & $(295)^3$ are equal to the विजाङ्क of given number, between them $(235)^3$ is best suited. $\therefore \sqrt[3]{12977875} = 235$

4.1.3. Illustration for Case III

The cube root of a number greater than 7 digits but ending with even number can be obtained by dividing the number whose cube root has to be extracted by 8 until odd cubs to be obtained, and can be used विजाङ्क method to ascertain the cube roots as in case II.

Example: Find $\sqrt[3]{5414689216}$.

Since, this is 10 digits even numbers we need to divide by 8 until odd cube to be obtained.

$$\begin{array}{r|l} 8 & 5414689216 \\ \hline 8 & 676836152 \\ \hline & 84604519 \end{array}$$

Here, the cube root of 84604519 can be find as in case II. i.e. $\sqrt[3]{84604519} = 439$.

Hence, $\sqrt[3]{5414689216} = \sqrt[3]{8 \times 8 \times (439)^3}$
$= 2 \times 2 \times 439 = 1756$.

5. CONCLUSION

It is very easy to find the cube roots of a number having less than 7 digits. But, it needs a little practice and patience initially to extract the cube roots of a number having more than 7 digits but less than 10 digits by Vedic methods. Also for the number greater than 7 digits but ending with even number, while extracting the cube root of a number having its unit digit, even we may get two values of M and it becomes difficult to ascertain the exact value of M, Vedic method provided the विजाङ्क method to determine it. It was not found the concrete proper and practical method to determine the cube roots of the

International Journal of Engineering, Pure and Applied Sciences,
Vol. 3, No. 1, March-2018

number before Aryabhtta's. All the Mathematician after him followed his method with some modifications. There is also symmetry between the method of Aryabhatta and Bharatikrishna Tirtharaj Marahaja to determine cube roots.

REFERENCES

- **Bathia, D. (2006).** *Vedic Mathematics Made Easy, Jaico Publishing House*: Mumbai, Bangalore.
- **Bhasin, S. (2005).** *Teaching of Mathematics a Practical Approach*, Himalaya publishing house, New Delhi.
- **Bose, S. (2014). Vedic Mathematics: V & S Publishers, Delhi.**
- **Chauthaiwala, M., Kolluru, R. (2010). Enjoy Vedic Mathematics: Sri Sri Publications Trust, Art of Living International Centre, Bangalore – 560082.**
- **Cutler, A. & Rudolph M**. *The Trachtenberg Speed System of Basic Mathematics (English edition), Asia Publishing House*, New Delhi, 2008
- **Glover, J.** T. (2002). Vedic Mathematics for schools: Motilal Banarasidass Publishers Pvt.Ltd.
- **Gupta, A.** (2006). The power of Vedic Mathematics: Jaico Publishing House 121 Mahatma Gandhi Road, Mumbai-400 001.
- **James, A. (2005).** *Teaching of Mathematics*: *Neelkamal Publications PVT. LTD.* Hyderabad.
- **Kapoor, S. K. (2006).Vedic Mathematics basics:** Lotus Press, Darya ganj, New Delhi-110002.
- **Kapoor, S. K. (2013). Learn and teach Vedic Mathematics:** Lotus Press, Daryaganj, New Delhi-110002.
- **Kapoor, S. K., Kapoor, R. P. (2010). Practice Vedic Mathematics (skills for perfection of intelligence) :** Lotus Press, Darya ganj, New Delhi-02.
- **Kumar, A. (2010).** Vedic *Mathematics Sutra*: *Upkar Prakasan*, Agra-2.
- **Mathomo, M. M. (2006)**. The *Use of Educational Technology in Mathematics Teaching and learning: An Investigation of a South African Rural Secondary School".*
- **Pant, Nayaraj (2037 B.S.):** "पण्डित गोपाल पाण्डे र उनको घनमूल ल्याउने रीति", नेपाल राजकीय प्रज्ञा प्रतिष्ठान
- **Satyamoorthi, H. M. (2012).** *Vedic Mathematics for Speed Arithmetic*, *Vasan Publications:* Bangalore.
- **Shalini, W. (2004).** *Modern Methods of Teaching Mathematics*, *Sarup & Sons*: New Delhi.
- **Sidhu, K. S. (1990)**. *The Teaching of Mathematics*, Sterling Publishers, Private Limited: New Delhi.
- **Singh, M. (2004).** Modern *teaching of Mathematics*, *Anmol Publications PVT LTD:* New Delhi.
- **Singhal, V.** (2014). Vedic Mathematics for all ages: Motilal Banarasidass Publishers Pvt. Ltd., Delhi.
- **Tirthaji B. K. (2009).** Vedic *mathematics*: *Motilal Banarsidass Publishers* Pvt. Ltd, Delhi.
- **Tirthaji**, **B. K. (1965).** *Vedic Mathematics*: *Motilal Banarasidass*, New Delhi, India.