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Satellites: Jacobian Integral of the equation of motion and elliptic orbit

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Abstract- This paper deals with the elliptic motion of center of mass, periodic Oscillation and it provides stable solution to the equation of oscillations. The change of frequencies of oscillation doesn't affect the stability of the Solution.

Keywords: stability; periodic oscillation; frequencies

1. INTRODUCTION

The effect of atmosphere resistance magnetic force and oblateness of the earth on the motion and stability of two satellites connected in the particular case of keplerian orbit that is circular orbit. The system moving in space has to face same perturbative force. These Perturbative forces compel the system which changes its orbit from circular to elliptic one.

2. MATHEMATICAL APPROCH

This system of equations of motion may be written in the following form

$$x'' - 2y' - 3x\rho + \frac{4A}{\rho}x + f\rho\rho' + \frac{B}{\rho}cosi = -\lambda\bar{\alpha}\left[1 - \frac{\ell o}{\rho_r}\right]x\rho^4$$

$$y'' + 2x' + f\rho^2 - {A}/{\rho}\, y + \frac{B\rho'}{\rho^2}cosi = -\overline{\lambda\alpha}\,\rho^4\left[1 - \frac{\ell o}{\rho_r}\right]y$$

$$z'' + z - \frac{A}{\rho}z = -\lambda\bar{\alpha}\rho^4\left[1 - \frac{\ell o}{\rho_r}\right]z\frac{-B}{\rho}\left[\frac{\rho'}{\rho}\cos(v+\omega) + \frac{1}{\mu_E}(3p^3\rho^2 - \mu_E)\sin(v+\omega)\right]sini \qquad ------(1)$$

Where,

$$r = \sqrt{x^2 + y^2 + z^2}$$

In case of elliptic orbit of the center of mass the dashes denote differentiation with respect to v of the orbit.

The condition of constraint takes the from

$$x^2 + y^2 + z^2 \leq \frac{\ell o^2}{\rho^2} \qquad -----(2)$$

The equations (1) are non-homogeneous and non-linear in character with the periodic forces .The appearance of the periodic effects on the motion of the satellite m_1 so long the period of oscillation of the system without their periodic force.

We assume that there periods are non-commensurate the equations of motion can be averaged with respect to v and the averaged equations of motion will describe the small perturbations and long periodic effects due to the periodic force on the motion of the satellite m_1.Now, we may obtain the averaged values the following terms :

$$\rho = \frac{1}{1+ecosv}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho dv = \frac{1}{(1-e^2)^{1/2}}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho\rho' dv = 0\,\frac{1}{2A}\int_0^{2A}\frac{\rho^1}{\rho^2}dv$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^2 dv = \frac{1}{(1-e^2)^{3/2}}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^3 dv = \frac{(2+e^2)}{2(1-e^2)^{5/2}} \qquad 3$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^4 dv = \frac{(2+3e^2)}{2(1-e^2)^{7/2}}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\rho^1}{\rho^2}\cos(v+\omega)\,dv = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^2\mathrm{sin}d(v+\omega)v = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{sin}d(v+\omega)v = 0$$

Here,

$$x'' - 2y' - \frac{3}{(1-e^2)^{1/2}}x + 4Ax + Bcosi = -\lambda\bar{\alpha}\left[\frac{(2+3e^2)}{2(1-e^2)^{7/2}} - \frac{(2+e^2)\ell o}{2r(1-e^2)^{5/2}}\right]x$$

$$y'' + 2x' + \frac{f}{(1-e^2)^{3/2}} - A.y = -\lambda\bar{\alpha}\left[\frac{(2+3e^2)}{2(1-e^2)^{7/2}} - \frac{(2+e^2)\ell o}{2r(1-e^2)^{5/2}}\right]y$$

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$$z'' + z - A.z = -\lambda\bar{\alpha}\left[\frac{(2+3e^2)}{2(1-e^2)^{7/2}} - \frac{(2+e^2)\ell o}{2r(1-e^2)^{5/2}}\right]z$$

Where

$$r^2 = x^2 + y^2 + z^2 \qquad ------(4)$$

∴ The system of equations given by (4) will represent the motion of the system while the relation

$$x^2 + y^2 + z^2 \leq \ell o^2(1-e^2)^{3/2} \quad ----------(5)$$

Gives the condition of the constraint we can say that the satellite m_1 is moving on the spare given by

$$x^2 + y^2 + z^2 = \ell o^2(1-e^2)^{3/2} ----------(6)$$

The equation (4) must exist Jacobi's integral for the system of equation (4) multiplying the three equations by 2x', 2y', 2z' and adding them together them after integrating

$$x'^2 + y'^2 + z'^2 - \left[\frac{3}{(1-e^2)^{3/2}} - 4A\right]x^2 - A.y^2 + z^2 - A.z^2 + 2Bx\cos i + \frac{2f}{(1-e^2)^{3/2}}y + \lambda\bar{\alpha}\frac{(2+3e^2)}{2(1-e^2)^{7/2}}(x^2+y^2+z^2) - \frac{\lambda\bar{\alpha}\ell o(2+e^2)}{(1-e^2)^{5/2}}[x^2+y^2+z^2]^{1/2} + he \quad ---------(7)$$

We have obtained the Jacobi's integral in the form of the equation (7) and *he* will be treated as the Jacobian constant. By obtaining the equation of the surface of zero velocity of the system .It follows that the satellite m_1 will be moving within the boundary of the surfaces of zero velocity.

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