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Minimizing Rental Cost in Two Stage Flow Shop Scheduling Problem Including Arbitrary Lags with Job Weightage

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Abstract- In this paper an attempt is made to study n jobs over two machines where processing times are associated with respective probabilities with time delays known as arbitrary lags i.e. start lag or stop lag. The objective of the study is to develop an efficient algorithm in order to minimize the rental cost with different system parameters such as arbitrary lags and job weightage as well. . Further jobs are associated with weights to indicate their preference in production scheduling .The proposed algorithm is easy to understand and also provide an important tool for the decision making in the field of production.

Keywords- Flowshop, Start Lag, Stop Lag, Transportation Time, Job weightage.

Subject classification number: 90B35, 90B36

1. INTRODUCTION

Scheduling is generally described as an allocation of a set of resources over time to perform a set of tasks. Scheduling emerges in various domains such as hospital management, airlines, trains, production scheduling etc. In other words scheduling refers to the second machine. The stop lag (Ei>, 0) for the job i is the minimum time which elapsed between the completion of it on second machine. we usually refer to *minimal* time lags, that is, the Minimal amount of time required to elapse between two consecutive placing jobs in a certain order or sequence so as to minimize the elapsed time with no passing between the jobs. By the time lag we meant the minimum time delay which is required between the executions of two consecutive operations of the same job. The start lag (Di>,0) is the minimum time which must elapse between starting job i on the first machine and starting it on operations of a job. In a feasible sequence, the actual time between two consecutive operations may be larger than the minimal time lag. A maximal time lag specifies an upper bound on the time delay between two consecutive operations of a job. The minimal (maximal) time lag between two consecutive operations of a job may be job-dependent or job-independent, machine-dependent or machine independent

Literature survey: laying the groundwork for scheduling, the genius of Johnson [1] is traced by considering his procedure for finding the optimal schedule for n-jobs and two machine flow-shop problem with taking account of objective of minimization of the make span (i.e. total elapsed time) . Also Mitten and Johnson [2] jointly discussed the n job, Although there are processing times but some additional tags are set up in 2 machine flowshop Scheduling problem. Maggu and Das [4] brought with themselves the concept of the equivalent job-block in the theory of scheduling which has many applications in the production management. Bagga [3], Maggu and Das [6],[7] Szwarch [8], Yoshida & Hitomi [5], Anup [13] etc. derived the optimal algorithm for two or multistage flow shop problems, considered the various constraints and criteria. Kern,W. ,Nawjin,W.M[10], J.Riezebos,G.J.C Goalman[11] continues with dealing different scheduling problems including time lags. Singh, T.P. and Gupta, D. [9],[12],[15][17] associated probabilities with processing time, set up time and transportation time as well as concept of breakdown interval in their studies. Singh, T.P, Gupta, D[13],[14] studied two /multiple flow shop problem to minimize rental cost under a policy which is predefined in which the probabilities are associated with processing time on each machine. The present paper is concerned with the flowshop scheduling problem in which processing times are associated with probabilities which includes arbitrary lags, transportation and job weightage as well for minimizing the rental cost for effective scheduling for better production.

Practical situations: some unavoidable circumstances in the business industry are occurred which do not offer sufficient funds to an entrepreneur to purchase machinery. Therefore, such situation forces him to take machine on rent or lease. So rental cost is indispensable part of schedule. Start lag and stop lag are the defined under the term time lag and is conducive in splitting and overlapping of jobs for effective and continuous running of machines. The

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application would be a situation in which each job is a batch consisting of several discrete / identical units. Once the first jobs are performed on a machine A it can immediately begin processing at machine B in that case, the start represents the time to process one unit on machine A and the stop lag represents the time to process one unit on machine 2.in other words, we would be using a transfer batch of size A so we can also model larger transfer batches with the use of time lags. in the case of start lags and stop lags, the optimal permutations schedule is characterized by a rule analogous to johnson's rule.

Theorem: Two-machine, n-job problem' with transportation times to given problem replacing three times (Start-lag, Stop-lag, transportation time) by single time t'_i

Proof: Let U_{ix} and T_{ix} denote Starting and Completion times of any job i on machine X (X = A, B, i = 1, 2, 3,…. n) respectively in a sequence S. From definition of Start-lag D_i, we have $U_{iB} - U_{iA} \geq D_i$

Now $T_{iA} - U_{iA} + A_i$

i.e.,Hence, we have, $U_{iB} - (T_{iA} - A_i) \geq D_i$

i.e.,$U_{iB} - T_{iA} \geq D_i - A_i$

… (1)

From definition of Stop-lag E_i,

we have ,$T_{iB} - T_{iA} \geq E_i$,

Now , $T_{iB} - U_{iB} + B_i$

Hence, we have $U_{iB} + B_i - T_{iA} \geq E_i$

i.e., $U_{iB} - T_{iA} \geq E_i - B_i$

… (2)

Also, from the definition of transportation time t_i, we have

$U_{iB} - T_{iA} \geq t_i$

… (3)

Let $t'_i = \max\{D_i - A_i, E_i - B_i, t_i\}$

… (4)

From (1), (2) and (3), it is obvious that

$U_{iB} - T_{iA} \geq t'_i$

… (5)

Notations

S_K : Sequence obtained by applying Johnson's Procedure.

a_i^1 : Processing time of i[th] job on machine A.

a_i^2 : Processing time of i[th] job on machine B.

A_α : Expected processing time of i[th] job on machine A.

B_α : Expected processing time of i[th] job on machine B.

p_i : Probability associated to a_i^1.

q_i : Probability associated to a_i^2.

β : Equivalent job for job-block.

W_i : Weight of i[th] job.

C_i : Rental cost of j[th] job.

D_i : Start lag for job i.

E_i : Stop lag for job i.

t_i : Transportation time of i[th] job.

t'_i : Effective transportation time of i[th].

$CT(S_k)$: Completion time of Sequence S_k.

$UT(S_k)$: Utilization time for which machine *Mj* is required

6. Algorithm

Step 1: Determine expected processing time given as follow:

$$A_\alpha = a_i^1 \times p_i$$
$$B_\alpha = a_i^2 \times q_i$$

Step 2: Determine effective transportation times given by:

$$t'_i = \max(D_i - A_\alpha, E_i - B_\alpha, t_i)$$

Step 3: Define two fictitious machines G & H with processing time G_i & H_i as below:

$$G_i = A_\alpha + t'_i$$
$$H_i = B_\alpha + t'_i$$

Step4: find out the weighted flowshop time G_i and H_i defined as follows:

If $\min(A_{i1}^1, A^1_{i2}) = A_{i1}^1$, then

$$G_i = \frac{(A_{i1}^1 + w_i)}{w_i} \text{ and } H_i = \frac{A_{i2}^1}{w_i}$$

And $\min(A_{i1}^1, A^1_{i2}) = A_{i2}^1$,then

$$G_i = \frac{A_{i1}^1}{w_i} \text{ and } H_i = \frac{(A_{i2}^1 + w_i)}{w_i}$$

Step5: Apply Johnson's (1954) technique to obtain the optimal Sequence S_K

.**Step 6:** Obtain each job having processing time on A greater than α. Put these job one by one in the first position of the sequence S_1 in the same order. Suppose these sequences be $S_2, S_3, S_4, \ldots S_r$.

Step 7. Construct In-Out table for each sequence Si (i=1,2,…r) and evaluate the total completion time for the last job of each sequence respectively.

Step 8. Evaluate completion time CT(Si) of first job for each sequence Si on machine A.

Step 9. Calculate utilization time Ui of second machine for each sequence Si as:

$U_i = t_2(S_i) - CT(S_i)$ for i=1,2,3,…….r.

Step10: Find Min {Ui}, i=1,2,…….r. let i=m the corresponding sequence, then Sm is the optimal sequence for minimum rental cost. Min rental cost = $t_1(S_m) \times C_1 + U_m \times C_2$.

Numerical illustration:

Obtain optimal sequence for 5 jobs and 2 machines problem given by the following tableau1:

Jobs	Machine A		Machine B		Transportation Time $T_{i,j} \to k$	Start Lag D_i	Stop Lag E_i	Job Weightage
	A_i	p_i	B_i	q_i				
1	18	.2	16	.1	4	15	22	5
2	16	.3	24	.2	7	18	18	6
3	14	.2	22	.3	8	20	22	8
4	22	.1	18	.2	7	16	10	7
5	20	.2	26	.2	8	24	19	8

Tableau 1

Our objective is to minimize the total rental cost of the machine where cost of machine A is 10 units and 20 units.

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Solution:

As Per Step 1: Define expected processing time A_{α}, B_{α} on both machines A and B respectively as shown in tableau-2

Jobs	Machine A_{α}	Machine B_{α}	Transportation Time $T_{i,j} \to k$	Start Lag D_i	Stop lag E_i	Job Weightage
1	3.6	1.6	4	15	22	5
2	4.8	4.8	7	18	18	6
3	2.8	6.6	8	20	22	8
4	2.2	3.6	7	16	10	7
5	4	5.2	8	24	19	8

Tableau 2

As Per Step 2: t_1^1= max (Di–Ai, Ei–Bi, ti)
= (15-3.6,22-1.6,4)=20.4

t_2^1= max (18-4.8,18-4.8,7)=13.2

t_3^1= max (20-2.8, 22-6.6,8) =17.2

t_4^1= max (16-2.2,10-3.6,6) =13.8

t_5^1= max (24-4, 19-5.2, 7) =20

so by above we find the expected transportations time as make fictitious machines that are shown in tableau 3

As Per Step 3:

Jobs	G_i	H_i	Job Weight age
1	24	22	5
2	18	18.4	6
3	20	23.8	8
4	16	17.4	7
5	24	25.2	8

Tableau 3

As per Step 4

Jobs	G_i	H_i
1	5.8	4.4
2	3	4.0
3	2.5	5.3
4	2.2	3.4
5	3	4.1

As per subject 5: By applying Johnson rule:

4	3	2	5	1

Tableau 4

As Per Step 6: We prepare in out table as shown in tableau 6

Jobs	Machine A IN-OUT	$T_j \to k$	Machine B IN-OUT
4	0-2.2	13.8	16-19.6
3	2.2-5	17.2	22.2-28.8
2	5-9.8	13.2	28.8-33.6
5	9.8-13.8	20	33.6-38.8
1	13.8-17.4	20.4	38.8-40.4

As per Step 7: The other optimal sequences for minimizing rental cost with utilization times are given in the form of table:

Sequences	Total time elapsed on machine A	Total time elapsed on machine B	Utilization Time
2,4,3,5,1	17.4	40.6	22.6
1,4,3,2,5	17.4	45.8	21.8
5,4,3,2,1	17.4	41.8	17.8
3,4,2,5,1	17.4	41.4	21.4

The total utilization of machine A is 17.4fixed units and minimum utilization of B is 17.8units for the sequence 5,4,3,2,1

Therefore minimum rental cost is = 17.4 x 10 + 17.8 x 20= 530 units

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